MATH 435 **Homework #3**

Spring 2009 **Prof. Biswa Nath Datta**

Due March 27, 2009

1. Derive the error formula for the standard Simpson's formula and using it prove the error formula for the composite Simpson's rule:

$$E_c^s = -\frac{(b-a)}{180} h^4 f^{(4)}(\eta),$$

where $a \leq \eta \leq b$. (Consult your textbook, if necessary).

2. Suppose that it is required to estimate $\int_0^1 e^{-x^2} dx$ within an accuracy of $\epsilon = 10^{-5}$. Find h using

   (a) The composite trapezoidal rule

   (b) The composite Simpson's rule

3. Let $P_n(x)$ be the nth Legendre polynomial and $Q(x)$ be any polynomial of degree less than n. Then prove that

$$\int_{-1}^{1} Q(x)P_n(x)dx = 0.$$

4. Approximate $\displaystyle\int_1^5 \frac{dx}{x+5}$ using Gaussian Quadrature with $n = 2$ and $n = 3$.

5. Prove that the Quadrature formula of the form

$$\int_1^3 f(x)dx \approx c_1 f(1) + c_2 f(2) + c_3 f(3)$$

that is exact for all polynomials of as high a degree as possible, is nothing but Simpson's rule.

6. Use adaptive Quadrature rule to approximate $\int_1^3 \frac{1}{x}\, dx$ to within $\epsilon = 10^{-3}$.

7. Suppose that Romberg integration is used to approximate

$$\int_0^1 \frac{x^2}{1+x^3} dx.$$

Develop a Romberg table with $n = 3$, and use this table to compute an approximation of the above integral.