(1) Let X be a compact Hausdorff space and x, y be two distinct points in X. Show that there is a continuous real-valued function f such that $f(x) \neq f(y)$. (In other words, the continuous real-valued functions on X *separate* the points of X.)

(2) Let X be a compact Hausdorff space and let $\{U_\alpha\}_{\alpha \in A}$ be an open covering of X. Show that there exists a finite number of continuous real-valued functions $h_1, \ldots, h_m$ on X that satisfy the following properties:
   (a) $0 \leq h_i \leq 1$, $1 \leq i \leq m$,
   (b) $\sum h_i = 1$,
   (c) For each i, there exists an index $\alpha_i \in A$ such that the closure of the set $\{x \mid h_i(x) > 0\}$ is contained in U_{α_i}.