MATH 550 TOPOLOGY HOMEWORK

DUE APRIL 22, 2005

(1) Let (Y, c) be a pointed topological space such that Y is locally path connected and simply connected, and let $p : (E, e) \to (X, b)$ be a covering map.
  (a) Show that every map $f : (Y, c) \to (X, b)$ can be uniquely lifted to a map $g : (Y, c) \to (E, b)$.
  (b) Suppose in addition that X is locally path connected and E is simply connected. Show that if f is a covering map, then the unique lift g is a homeomorphism.

(2) Show that any two-sheeted covering admits a unique G-covering structure.

(3) Show that S^n is simply connected for $n \geq 2$.

(4) Let X be the figure-eight and $p : E \to X$ be the three-sheeted covering introduced in class. What is the group $\mathrm{Aut}(E/X)$ of covering transformations?

(5) Let G be the subgroup of the group of homeomorphisms of the plane to itself generated by $(x, y) \mapsto (x + 1, y)$ and $(x, y) \mapsto (-x, y + 1)$.
  (a) Show that the G action on $\mathbb{R}^2$ is even.
  (b) What is the quotient space $\mathbb{R}^2/G$?