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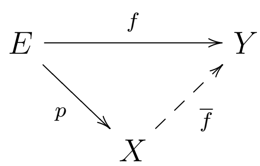
MATH 550 TOPOLOGY HOMEWORK

DUE 12PM ON MAY 8, 2005

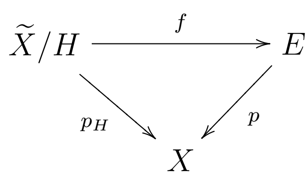
(1) A covering map $p : E \to X$ is said to be *regular* if $p_*\pi_1(E, e)$ is a normal subgroup of $\pi_1(X, x)$. Show that a G-covering $p : E \to X$ is regular if E is connected and locally path-connected.

(2) Show that a simply connected and locally path-connected space has only trivial coverings.

(3) Suppose that a topological group G acts on E properly discontinuously, and let $p : E \to X := E/G$ be the canonical projection map to the orbit space. Prove that any G-invariant continuous map $E \to Y$ factors through X. i.e. for any map $f : E \to Y$ such that $f(g \cdot e) = f(e)$ for all $g \in G$ and $e \in E$, there is a map $\overline{f} : X \to Y$ such that $\overline{f} \circ p = f$.



(4) Let X be a topological space and $\pi : \widetilde{X} \to X$ be a universal covering of X. Let $H \subset \pi_1(X)$ be a subgroup and $p : E \to X$ be a connected covering map such that $p_*\pi_1(E) = H$. Show that there is a canonical homeomorphism $f : \widetilde{X}/H \to E$ such that the following diagram is commutative:



[1]Solution will be posted on May 8 at 5PM.