Groups Part II

We have seen several examples of groups. Let's look at some examples of sets which **aren't** groups.

One way to combine two vectors in $\mathbb{R}^3$ is to take their dot product:

$$< x_1, x_2, x_3 > \cdot < y_1, y_2, y_3 >= x_1y_1 + x_2y_2 + x_3y_3.$$

But $\mathbb{R}^3$ with this dot product is not a group, since the dot product of two vectors isn't another vector.

Another way to combine two vectors in $\mathbb{R}^3$ is to take their cross product:

$$< x_1, x_2, x_3 > \times < y_1, y_2, y_3 >=< x_2y_3 - y_2x_3, y_1x_3 - x_1y_3, x_1y_2 - y_1x_2 > .$$

At least this way of combining two vectors in $\mathbb{R}^3$ gives us another vector in $\mathbb{R}^3$. Is $\mathbb{R}^3$ with the cross product a group?

Is the cross product associative? In other words, is $\mathbf{x} \times (\mathbf{y} \times \mathbf{z}) = (\mathbf{x} \times \mathbf{y}) \times \mathbf{z}$ for any three vectors $\mathbf{x}, \mathbf{y}$ and $\mathbf{z}$?

Is there an "identity" vector? In other words, is there a special vector $\mathbf{v}$ where $\mathbf{x} \times \mathbf{v} = \mathbf{v} \times \mathbf{x} = \mathbf{x}$ for all vectors $\mathbf{x}$?

More examples to look at are 1a, 1c, 1e, and 8 on page 90.