Amazing
Powers
of
COMPUTATION

A number is divisible by 3 (or 9) if and only if it is congruent to 0 modulo 3 (or 9).
When we write a number, we write in base 10:

\[ 3875 = 3 \cdot 10^3 + 8 \cdot 10^2 + 7 \cdot 10^1 + 5 \cdot 10^0. \]

Therefore, it is congruent to a combination of the digits:

\[ 3875 \equiv 3 \cdot 10^3 + 8 \cdot 10^2 + 7 \cdot 10 + 5 \pmod{3} \]
\[ \equiv 3 \cdot 1 + 8 \cdot 1 + 7 \cdot 1 + 5 \pmod{3} \]
\[ \equiv 3 + 8 + 7 + 5 \pmod{3}, \]

since all the powers of 10 are congruent to 1 modulo 3:

\[ 10 \equiv 1 \pmod{3}, \quad 10^2 \equiv 1^2 \pmod{3}, \quad 10^3 \equiv 1^3 \pmod{3} \ldots \]

Suppose we try this “trick” with 9. What are 10, 10^2, 10^3, etc. congruent to modulo 9?

What is the “trick” to seeing if a number is divisible by 9?

What are the powers of 10 congruent to modulo 11?

What is the “trick” to seeing if a number is divisible by 11?

What about other numbers such as 2, 4, 5 and 7?