1. Graph the ellipse $4x^2 + 9y^2 = 36$. Label vertices and foci.

   Sorry, but putting graphs in these pdf files is a real pain. To find the vertices and foci, write
   
   $$\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1.$$ 

   The vertices are $(\pm 3, 0)$. To find the foci, use the $c^2 = a^2 - b^2$ formula. The foci are $(\pm \sqrt{5}, 0)$.

2. Find an equation for the tangent line to the curve given parametrically by $x = t^3 - e^t$, $y = \ln t + t$ at the point where $t = 2$.

   To find the slope, use $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$. In our case
   
   $$\frac{dy}{dx} = \frac{1 + (1/t)}{3t^2 - e^t} = \frac{3/2}{12 - e^2}$$

   at the point where $t = 2$. This point is $(8 - e^2, 2 + \ln 2)$. In point-slope form, the tangent line is given by
   
   $$y - (2 + \ln 2) = \frac{3/2}{12 - e^2} (x - (8 - e^2)).$$

3. Find the area of the triangle with vertices $P(1, 2, 3)$, $Q(2, 3, 1)$ and $R(3, 1, 2)$.

   The vector from $P$ to $Q$ is $<1, 1, -2>$ and the vector from $P$ to $R$ is $<2, -1, -1>$. The cross product is
   
   $$\langle -1 - 2, -4 - (-1), -1 - 2 \rangle = \langle -3, -3, -3 \rangle.$$

   The length of the cross product is
   
   $$\sqrt{3^2 + 3^2 + 3^2} = 3\sqrt{3}.$$

   The area of the triangle is half of this, $3\sqrt{3}/2.$
4. Set up, but do not evaluate, an integral to compute the arclength of one leaf of the rose $r = 2\sin(2\theta)$. Include a sketch of the curve.

Sorry, no sketch here. It’s a four-leafed rose. Using $x = r \cos \theta$ and $y = r \sin \theta$, you get $dx/d\theta = 4 \cos(2\theta) \cos \theta - 2 \sin(2\theta) \sin \theta$ and $dy/d\theta = 4 \cos(2\theta) \sin \theta + 2 \sin(2\theta) \cos \theta$. The arclength is

$$\int_{\alpha}^{\beta} \sqrt{(dx/d\theta)^2 + (dy/d\theta)^2} \, d\theta.$$ 

To get one leaf of the rose, you start at $\theta = 0$ and go to $\theta = \pi/2$. So those are the limits of integration.

5. Describe the surface given by the equation $r = z$ in cylindrical coordinates.

You can, if you wish, rewrite this in Cartesian coordinates as $x^2 + y^2 = z^2$. This surface is two right circular cones centered on the $z$-axis and meeting at the origin.

6. Find the distance from the point $P(1, 2, 3)$ to the plane given by $x - 3y + 7z = 4$.

Pick a point on this plane and call it $Q$. I’ll choose $(4, 0, 0)$. Let $\mathbf{a}$ be the vector from $P$ to $Q$: $\mathbf{a} = (-3, 2, 3)$. A normal vector to this plane is $\mathbf{n} = (1, -3, 7)$. The distance from $P$ to the plane is the absolute value of the scalar component of $\mathbf{a}$ in the direction $\mathbf{n}$. In our case, it’s

$$\frac{|\mathbf{a} \cdot \mathbf{n}|}{|\mathbf{n}|} = \frac{|-3 - 6 + 21|}{\sqrt{1 + 9 + 49}} = \frac{12}{\sqrt{59}}.$$