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1. Find an equation for the tangent line to the graph of $y = x^3 - x$ at the point where $x = 1$. Graph this function together with the tangent line.

First find the slope. Setting $f(x) = y$ and noting that $f(1) = 0$, the slope is

$$\begin{aligned}\lim_{x\to 1} \frac{f(x) - f(1)}{x - 1} &= \lim_{x\to 1} \frac{x^3 - x}{x - 1} \\ &= \lim_{x\to 1} \frac{x(x-1)(x+1)}{x - 1} \\ &= \lim_{x\to 1} x(x+1) \\ &= 2.\end{aligned}$$

In point-slope form, an equation for the tangent line is

$$y - 0 = 2(x - 1).$$

You're on your own with the graphs.

2. Suppose

$$f(x) = \begin{cases} -x & \text{if } x < 0, \\ 1 & \text{if } 0 < x < 1, \\ \frac{3-x}{2} & \text{if } x \geq 1. \end{cases}$$

Graph this function. At what point(s), if any, is this function not continuous? At what points(s), if any, is this function not differentiable? Explain.

This is actually from a homework exercise; this graph is #25a from §1.1 As you see from the graph, it is continuous everywhere except at $x = 0$, where there is a break in the graph (and the function is undefined). It is also not differentiable there, since a function must be continuous to be differentiable. Moreover, it is also not differentiable at $x = 1$ since the graph has an abrupt change of direction there.

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3. Show that the equation $x^4 - 3x^2 + x - 1 = 0$ has at least one root.

You were supposed to use the Intermediate Value Theorem here. In this case, the function $y = x^4 - 3x^2 + x - 1$ is continuous everywhere. We just need to find one place where it's negative and another place where it's positive. For example, $f(0) = -1$ and $f(2) = 16 - 12 + 2 - 1 = 5$. So $f(x) = 0$ for at least one value of x between 0 and 2.

4. Find a number $\delta > 0$ such that $|(4x - 3) - 5| < .01$ whenever $0 < |x - 2| < \delta$. With what limit would this be associated?

"Solving" the inequality, you have

$$|(4x - 3) - 5| < .01$$

$$|4x - 8| < .01$$

$$4|x - 2| < .01$$

$$|x - 2| < .0025.$$

So any value of δ no larger than .0025 works. The associated limit is

$$\lim_{x \to 2} 4x - 3 = 5.$$

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5. Evaluate the following limits. Be sure to show all steps.

a) $\lim_{x \to 2} \dfrac{x^4 - 16}{x + 2}$

Just "plug in" $x = 2$; you get

$$\lim_{x \to 2} \frac{x^4 - 16}{x + 2} = \frac{0}{4} = 0.$$

b) $\lim_{x \to 0^+} \dfrac{\sqrt{2x + 4} - 2}{x}$

Multiplying by the conjugate, you get

$$\begin{aligned}
\lim_{x \to 0^+} \frac{\sqrt{2x+4} - 2}{x} &= \lim_{x \to 0^+} \frac{\sqrt{2x+4} - 2}{x} \times \frac{\sqrt{2x+4} + 2}{\sqrt{2x+4} + 2} \\
&= \lim_{x \to 0^+} \frac{2x + 4 - 4}{x(\sqrt{2x+4} + 2)} \\
&= \lim_{x \to 0^+} \frac{2}{(\sqrt{2x+4} + 2)} \\
&= \frac{2}{\sqrt{4} + 2} \\
&= \frac{1}{2}.
\end{aligned}$$

c) $\lim_{x \to -\infty} \dfrac{2x^3 - 3x}{x^3 + 10x^2 + 100}$

Divide the numerator and denominator by the highest power of x, which is x^3. This limit is equal to

$$\begin{aligned}
\lim_{x \to -\infty} \frac{2 - (3/x^2)}{1 + (10/x) + (100/x^3)} &= \frac{2 - 3\lim_{x \to -\infty}(1/x^2)}{1 + 10\lim_{x \to -\infty}(1/x) + 100\lim_{x \to -\infty}(1/x^3)} \\
&= \frac{2 - 0}{1 + 0 + 0} \\
&= 2.
\end{aligned}$$

d) $\lim_{x \to 3^-} \dfrac{x^2 + 9}{(x - 3)^2}$

Here the numerator approaches $3^2 + 9 = 18$. The denominator is always positive (since it is a square), so approaches 0 *from above.* Thus, the limit here is ∞.