Math 229 Section 10 Quiz #10 Solutions

1. Evaluate the integral $\int_{-3}^{4} \left(5 - \frac{x}{2}\right) dx$

You can either graph this and compute the integral using areas or use antiderivatives. I'll go low tech and use the first option. The graph of $y = 5 - \frac{x}{2}$ on $[-3, 4]$ is a line segment starting at $(-3, 13/2)$ and ending at $(4, 3)$. The integral gives the area under the graph, which is the area of a trapezoid with base width 7, large height 13/2 and small height 3. The average height is $\left(\frac{13}{2} + 3\right)/2 = 19/4$, so the total area is $7 \times 19/4 = 133/4$.

2. Evaluate the integral $\int_{-4}^{4} |x|\, dx$

Again you can do this via areas. When you graph this, you see that the integral is the area of two right triangles with base width 4 and height 4, so total area 16. (Each triangle has area 8.) Another option is to rewrite the integral to get rid of the absolute values:

$$\begin{aligned} \int_{-4}^{4} |x|\, dx &= \int_{-4}^{0} |x|\, dx + \int_{0}^{4} |x|\, dx \\ &= \int_{-4}^{0} -x\, dx + \int_{0}^{4} x\, dx. \end{aligned}$$

You get this since $|x| = -x$ for x in the interval $[-4, 0]$ and $|x| = x$ for x in the interval $[0, 4]$. Evalutating these integrals via anti-derivatives gives you the same answer, of course.

3. Suppose that f has a positive derivative for all values of x and that $f(1) = 0$. Which of the following statements must be true of the function

$$g(x) = \int_{0}^{x} f(t)\, dt?$$

a. g has a local maximum at $x = 1$.
b. The graph of g has an inflection point at $x = 1$.
c. The graph of dg/dx crosses the x-axis at $x = 1$.

The important thing here is to use the Fundamental Theorem of Calculus to see that $g'(x) = f(x)$. In particular, $g''(x) = f'(x)$ which we are told is positive. So g is always concave up and has no local maxima or points of inflection. Also, $g'(1) = f(1) = 0$, so the graph of dg/dx does cross the x-axis as $x = 1$.