Math 229 Section 10 Quiz #9 Solutions

1. Find the indefinite integral $\int \left(3t^2 + \frac{t}{2}\right) dt$. Check your answer by differentiation.

By the sum and constant multiple rules,

$$\begin{aligned}\int \left(3t^2 + \frac{t}{2}\right) dt &= \int 3t^2 \, dt + \int \frac{t}{2} \, dt \\ &= t^3 + \frac{1}{4} \int 2t \, dt \\ &= t^3 + \frac{1}{4} t^2 + C.\end{aligned}$$

Checking this by differentiating via the sum and constant multiple rules:

$$\begin{aligned}\frac{d\left(t^3 + \frac{1}{4}t^2\right)}{dt} &= \frac{dt^3}{dt} + \frac{1}{4}\frac{dt^2}{dt} \\ &= 3t^2 + \frac{1}{2}t.\end{aligned}$$

2. Solve the initial value problem $dy/dx = 2x - 7, \; y(2) = 0$.

First,

$$\begin{aligned}y = \int 2x - 7 \, dx &= \int 2x \, dx - \int 7 \, dx \\ &= x^2 - 7 \int 1 \, dx \\ &= x^2 - 7x + C.\end{aligned}$$

Next, find C by plugging in $x = 2$:

$$0 = y(2) = 2^2 - 7(2) + C, \qquad C = 10.$$

So $y = x^2 - 7x + 10$.

3. Use an upper sum with four rectangles of equal width to approximate the area under the graph of $y = 1/x$ between $x = 1$ and $x = 5$.

Here "upper sum" means an overestimate, so in this case you should use rectangles whose heights are coming from left endpoints. The base of each rectangle is 1 and the heights are $1, \; 1/2, \; 1/3$ and $1/4$. The sum of the areas of the four rectangles is $1 + 1/2 + 1/3 + 1/4 = 25/12$. This approximates the area under the graph.