

Exercises for Math 480 Week #10

1. For any real number  $\alpha$  and integer  $n$ , show that  $\alpha + n \sim \alpha$ . If  $\alpha \neq 0$ , show that  $\alpha \sim 1/\alpha$ .
2. Find the continued fraction expansions for  $\sqrt{2}$  and  $\frac{\sqrt{2}+3}{7}$ . Conclude that they are equivalent, and find  $A, B, C, D \in \mathbb{Z}$  with  $AD - BC = \pm 1$  and

$$\frac{\sqrt{2} + 3}{7} = \frac{A\sqrt{2} + B}{C\sqrt{2} + D}.$$

3. Assuming that

$$e - 1 = [1; 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, \dots],$$

show that  $e$  is *not* a root of any non-zero polynomial of the form  $aX^2 + bX + c$ , where  $a, b, c \in \mathbb{Z}$ .

4. Find the continued fraction expansions of  $\sqrt{7}$ ,  $\sqrt{10}$ ,  $\sqrt{11}$  and  $\sqrt{13}$ . What are the periods of each? To which purely periodic number is each of these equivalent?

5. Find the continued fraction expansions of  $\sqrt{5}$ ,  $\sqrt{17}$  and  $\sqrt{26}$ . You've already found  $\sqrt{2}$  and  $\sqrt{10}$ . Make a general conjecture about the continued fraction expansion of  $\sqrt{n^2 + 1}$ . Can you prove your conjecture? What about  $\sqrt{n^2 - 1}$ ?