

Math 229 Section 1 Quiz #2 Solutions

1. Given that

$$\lim_{x \rightarrow 2} f(x) = 4, \quad \lim_{x \rightarrow 2} g(x) = -2, \quad \lim_{x \rightarrow 2} h(x) = 0,$$

find $\lim_{x \rightarrow 2} \frac{g(x)h(x)}{f(x)}$ or, if it doesn't exist, explain why.

Use your limit laws:

$$\lim_{x \rightarrow 2} \frac{g(x)h(x)}{f(x)} = \frac{\lim_{x \rightarrow 2} g(x) \lim_{x \rightarrow 2} h(x)}{\lim_{x \rightarrow 2} f(x)} = \frac{-2 \cdot 0}{4} = 0.$$

(This is okay since the *denominator* isn't 0.)

2. Sketch the graph of a function f that satisfies all of the given conditions:

$$\begin{aligned} \lim_{x \rightarrow 0^-} f(x) = 1, & \quad \lim_{x \rightarrow 0^+} f(x) = -1, & \quad \lim_{x \rightarrow 2^-} f(x) = 0, \\ \lim_{x \rightarrow 2^+} f(x) = 1, & \quad f(2) = 1, & \quad f(0) \text{ is undefined.} \end{aligned}$$

Sorry, you're on your own here since putting pictures in these files is a royal pain. That said, you can see graphs of similar situations in the book. For example, figure 10 on page 71 shows two different one-sided limits and no function value.

3. If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height in meters t seconds later is given by $y = 10t - 1.86t^2$. Find the average velocity over the time intervals $[1, 1.1]$, $[1, 1.101]$ and $[1, 1.001]$. Estimate the instantaneous velocity when $t = 1$.

First, when $t = 1$ you have $y = 10 - 1.86 = 8.14$. Now the average velocity over a time interval from 1 to t will be the quotient $\frac{10t - 1.86t^2 - 8.14}{t - 1}$. You can type this into your calculator and then just plug in the various values of t requested. I get 6.094 when $t = 1.1$, 6.0921 when $t = 1.101$ (whoops! that was a typo; it should be $[1, 1.01]$), and 6.2781 when $t = 1.001$. That's probably not enough information to make an educated guess for the instantaneous velocity. After trying more values of t closer to 1, a reasonable guess is 6.28. In fact, that's what my calculator gives once I'm as close as 1.00001.