

Math 229 Section 1 Quiz #3 Solutions

1. Evaluate the limit, if it exists:

$$\lim_{x \rightarrow 4} \frac{x^2 - 4x}{x^2 - 3x - 4}$$

You're supposed to factor the numerator and denominator here:

$$\frac{x^2 - 4x}{x^2 - 3x - 4} = \frac{x(x - 4)}{(x + 1)(x - 4)}$$

Using this, you see that $\frac{x^2 - 4x}{x^2 - 3x - 4} = \frac{x}{x + 1}$ except when $x = 4$. Luckily we don't care about $x = 4$ with our limit (see page 81); in other words,

$$\lim_{x \rightarrow 4} \frac{x^2 - 4x}{x^2 - 3x - 4} = \lim_{x \rightarrow 4} \frac{x}{x + 1} = \frac{4}{5}$$

(You can confirm this with careful use of your calculator, as shown in class.)

2. Suppose f and g both have domain all the real numbers and that $\lim_{x \rightarrow 0} f(x)/g(x) = 1$. Is it possible for $f(0) = 0$? Can $g(0) = 0$?

Sure it's possible. In fact, suppose $f(x) = g(x) = x$. Then both of them are 0 at $x = 0$ (and the limit is 1, of course).

3. Is it true that a continuous function that is never 0 is either always positive or always negative? Give reasons for your answer.

Yes, this is true. To see why, suppose not; say $f(a)$ is positive and $f(b)$ is negative for some a and b . Then $f(b) < 0 < f(a)$ and by the Intermediate Value Theorem $f(x) = 0$ for some x between a and b .