Math 229 Section 1 Quiz #4 Solutions

1. Find the limit $\lim_{x \to 0} \dfrac{\sin 4x}{\sin 6x}$

To evaluate this, you do some algebra to see $\lim_{\theta \to 0} \frac{\sin \theta}{\theta}$:

$$\begin{aligned}
\lim_{x \to 0} \frac{\sin 4x}{\sin 6x} &= \lim_{x \to 0} \frac{\frac{4x \sin 4x}{4x}}{\frac{6x \sin 6x}{6x}} \\
&= \frac{4}{6} \lim_{x \to 0} \frac{\frac{\sin 4x}{4x}}{\frac{\sin 6x}{6x}} \\
&= (2/3) \frac{\lim_{\theta \to 0} \frac{\sin \theta}{\theta}}{\lim_{\phi \to 0} \frac{\sin \phi}{\phi}} \qquad (\theta = 4x,\ \phi = 6x) \\
&= (2/3) \frac{1}{1} = 2/3.
\end{aligned}$$

2. Find $h'(2)$, given that $f(2) = -3,\ g(2) = 4,\ f'(2) = -2$, and $g'(2) = 7$.
a) $h(x) = f(x)g(x)$

By the product rule,

$$h'(2) = f'(2)g(2) + f(2)g'(2) = (-2)4 + (-3)7 = -29.$$

b) $h(x) = \dfrac{g(x)}{1 + f(x)}$

By the quotient rule and the sum rule (which gives $\frac{d(1+f(x))}{dx} = \frac{d1}{dx} + f'(x) = f'(x)$),

$$h'(2) = \frac{g'(2)\big(1 + f(2)\big) - g(2)f'(2)}{(1 + f(2))^2} = \frac{7(1-3) - 4(-2)}{(1-3)^2} = \frac{-6}{4}.$$

3. Find an equation for the tangent line to the curve $y = x^4 + 2x^2 - x$ at the point $(1, 2)$.

The slope is given by the derivative, and

$$\frac{dy}{dx} = \frac{dx^4}{dx} + \frac{d2x^2}{dx} - \frac{dx}{dx} = 4x^3 + 2\frac{dx^2}{dx} - 1 = 4x^3 + 4x - 1.$$

At the point where $x = 1$, the slope is $4 \cdot 1^3 + 4 \cdot 1 - 1 = 7$. In point-slope form, an equation for the tangent line is

$$(y - 2) = 7(x - 1).$$