Math 229 Section 1 Quiz #5 Solutions

1. Find an equation for the tangent line to the curve $y = (1+x)\cos x$ at the point $(0,1)$.

The slope of the tangent line is given by the derivative. Using the product rule then the sum rule:

$$\begin{aligned}\frac{dy}{dx} &= \frac{d(1+x)}{dx}\cos x + (1+x)\frac{d\cos x}{dx}\\ &= \left(\frac{d1}{dx} + \frac{dx}{dx}\right)\cos x + (1+x)(-\sin x)\\ &= \cos x - (1+x)\sin x.\end{aligned}$$

The slope of the tangent when $x = 0$ is $y'(0) = \cos 0 - (1+0)\sin 0 = 1$. Using point-slope form, an equation for the tangent line is

$$y - 1 = 1(x - 0).$$

2. Find the derivative of $y = x\sin\sqrt{x}$.

Using the product rule first, then the chain rule with $u = \sqrt{x} = x^{1/2}$,

$$\begin{aligned}\frac{dy}{dx} &= \frac{dx}{dx}\sin\sqrt{x} + x\frac{d\sin\sqrt{x}}{dx}\\ &= \sin\sqrt{x} + x\frac{d\sin u}{du}\ \frac{dx^{1/2}}{dx}\\ &= \sin\sqrt{x} + x\cos(u)\ (1/2)x^{-1/2}\\ &= \sin\sqrt{x} + x\cos(\sqrt{x})\ (1/2)x^{-1/2}.\end{aligned}$$

3. Find dy/dx by implicit differentiation when $2x^3 + x^2y - xy^3 = 2$.

Using the sum/difference rule, constant multiple rule, product rule and chain rule,

$$\begin{aligned}\frac{d(2x^3 + x^2y - xy^3)}{dx} &= \frac{d2}{dx}\\ \frac{d2x^3}{dx} + \frac{dx^2y}{dx} - \frac{dxy^3}{dx} &= 0\\ 2\frac{dx^3}{dx} + \frac{dx^2}{dx}y + x^2\frac{dy}{dx} - \left(\frac{dx}{dx}y^3 + x\frac{dy^3}{dx}\right) &= 0\\ 2(3x^2) + 2xy + x^2\frac{dy}{dx} - y^3 - x\frac{dy^3}{dy}\frac{dy}{dx} &= 0\\ 6x^2 + 2xy + x^2\frac{dy}{dx} - y^3 - x(3y^2)\frac{dy}{dx} &= 0.\end{aligned}$$

Solving for $\frac{dy}{dx}$, you get

$$\begin{aligned}(x^2 - 3xy^2)\frac{dy}{dx} &= -6x^2 - 2xy + y^3 \\ \frac{dy}{dx} &= \frac{-6x^2 - 2xy + y^3}{x^2 - 3xy^2}.\end{aligned}$$