**Math 229 Section 1 Quiz #6 Solutions**

1. Sketch the graph of a function $f$ that is continuous on [1,5], has no local maximum or minimum, but 2 and 4 are critical numbers.

No pictures here. One way to avoid local extrema would be to have sharp "corners" on your graph (where there would be no derivative). So one way to draw your graph is with 3 line segements with all positive slope or all negative slope, connecting at $x = 2$ and 4. Another way to avoid local extrema is with a horizontal tangent such as with $y = x^3$ at $x = 0$, or with a vertical tangent such as with $y = \sqrt[3]{x}$ at $x = 0$.

2. Find the critical numbers of the function $h(p) = \dfrac{p-1}{p^2+4}$.

Take the derivative of $h$ using the quotient rule:

$$\begin{aligned} h'(p) &= \frac{(p^2+4)\frac{d(p-1)}{dp} - (p-1)\frac{d(p^2+4)}{dp}}{(p^2+4)^2} \\ &= \frac{(p^2+4)(dp/dp - d1/dp) - (p-1)(dp^2/dp + d4/dp)}{(p^2+4)^2} \\ &= \frac{(p^2+4) - (p-1)2p}{(p^2+4)^2} \\ &= -(p^2-2p-4)(p^2+4)^{-2}. \end{aligned}$$

You can find the roots of $p^2 - 2p - 4$ using the quadratic formula; the are

$$p = \frac{2 \pm \sqrt{4+16}}{2} = 1 \pm \sqrt{5}.$$

These are the critical numbers, since $p^2 + 4$ has no roots.

3. Find the absolute maximum and absolute minimum values of $f(x) = x^3 - 6x^2 + 9x + 2$ on $[-1, 4]$.

First find any critical numbers in $[-1, 4]$. The derivative of $f$ is

$$f'(x) = \frac{dx^3}{dx} - \frac{d6x^2}{dx} + \frac{d9x}{dx} + \frac{d2}{dx} = 3x^2 - 6\frac{dx^2}{dx} + 9\frac{dx}{dx} = 3x^2 - 12x + 9.$$

In order to find any critical numbers, we need to factor this: $f'(x) = 3(x^2-4x+3) = 3(x-1)(x-3)$. So 1 and 3 are critical numbers in our interval. Plugging these and the endpoints into $f$ gives us four functions values: $f(-1) = -1-6-9+2 = -14$, $f(1) = 1-6+9+2 = 6$, $f(3) = 27-54+27+2 = 2$ and $f(4) = 64 - 96 + 36 + 2 = 6$. The absolute minimum value is $-14$ and the absolute maximum value is 6.