Math 229 Section 1 Quiz #9 Solutions

1. Estimate the area under the graph of $f(x) = \sqrt{x}$ from $x = 0$ to $x = 4$ using four approximating rectangles and right endpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?

The four rectangles, assuming you make them all the same width, will have a width of 1 and the four right endpoints are 1, 2, 3 and 4. To get the heights, you use these numbers in the function, getting 1, $\sqrt{2}$, $\sqrt{3}$ and 2. The sum of the areas of the four rectangles is $1+\sqrt{2}+\sqrt{3}+2$ (the areas are just the heights, since the widths are all 1). A graph will clearly show that these rectangles extend above the graph except at the right upper corner, so this sum is an overestimate for the area under the graph. (Give your answer to four decimal places.)

2. Use Newton's method with $x_1 = -1$ to find x_3, the third approximation to the root of the equation $x^5 + 2 = 0$.

Setting $f(x) = x^5 + 2$, you get $f'(x) = 5x^4$. In this case the formula for Newton's method is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^5 + 2}{5x_n^4}.$$

Starting with $x_1 = -1$, we have

$$x_2 = x_1 - \frac{x_1^5 + 2}{5x_1^4} = -1 - \frac{-1+2}{5} = -6/5$$

and

$$x_3 = x_2 - \frac{x_2^5 + 2}{5x^4} = -6/5 - \frac{-(6/5)^5 + 2}{5(6/5)^4} = -6/5 - \frac{2 \cdot 5^5 - 6^5}{5^2 6^4} = \frac{-6^5 \cdot 5 - 2 \cdot 5^5 + 6^5}{5^2 6^4} = -\frac{37354}{32400},$$

which is approximately -1.1529.

3. Find f if $f'(x) = 6x + \sin x$.

An antiderivative of x is $x^2/2$ and an antiderivative of $\sin x$ is $-\cos x$, so using the sum and constant multiple rules, an antiderivative of $6x + \sin x$ is $3x^2 - \cos x$. So one answer is $f(x) = 3x^2 - \cos x$. The general antiderivative is $3x^2 - \cos x + C$, so you can add any number you like to $3x^2 - \cos x$.