

Groups Part II

We have seen several examples of groups. Let's look at some examples of sets which **aren't** groups.

One way to combine two vectors in \mathbb{R}^3 is to take their dot product:

$$\langle x_1, x_2, x_3 \rangle \cdot \langle y_1, y_2, y_3 \rangle = x_1y_1 + x_2y_2 + x_3y_3.$$

But \mathbb{R}^3 with this dot product is not a group, since the dot product of two vectors isn't another vector.

Another way to combine two vectors in \mathbb{R}^3 is to take their cross product:

$$\langle x_1, x_2, x_3 \rangle \times \langle y_1, y_2, y_3 \rangle = \langle x_2y_3 - y_2x_3, y_1x_3 - x_1y_3, x_1y_2 - y_1x_2 \rangle .$$

At least this way of combining two vectors in \mathbb{R}^3 gives us another vector in \mathbb{R}^3 . Is \mathbb{R}^3 with the cross product a group?

Is the cross product associative? In other words, is $\mathbf{x} \times (\mathbf{y} \times \mathbf{z}) = (\mathbf{x} \times \mathbf{y}) \times \mathbf{z}$ for any three vectors \mathbf{x}, \mathbf{y} and \mathbf{z} ?

Is there an "identity" vector? In other words, is there a special vector \mathbf{v} where $\mathbf{x} \times \mathbf{v} = \mathbf{v} \times \mathbf{x} = \mathbf{x}$ for all vectors \mathbf{x} ?