Groups Part III

Here are three more examples of groups.

1) The set of even integers (usually denoted $2\mathbb{Z}$) with addition.

(Please note that this is most definitely *not* $\mathbb{Z}_2$.)

2) The set of even permutations in S_4 with composition.

More generally, the set of even permutations in S_n for any n.

(These groups are usually denoted A_n and are called the *alternating group* on n letters.)

3) The set of upper triangular and invertible 2×2 matrices with matrix multiplication.

In these examples, did we need to check for associativity?

Is there any way to make your life easier when you want to check if a particular subset of a known group is a group?

Definition: A *subgroup* of a group G is a subset of G which is a group in its own right, using the same binary operation.

Is it possible for a subgroup of a group to have a new and different identity element? In other words, if G is a group with identity element e and H is a subgroup of G, then H has an identity element, too. Must the identity element of H be e?

Generally speaking, in order for a subset H of a group G to be a subgroup, we must be sure that

- H is closed: if a and b are elements of H, then so is ab.
- The identity element e of G is in H.
- For every element $a \in H$, a^{-1} is an element of H, too.

Corollary 3.2.3: Let G be a group and H be a non-empty subset of G. Then H is a subgroup if and only if $ab^{-1} \in H$ whenever $a, b \in H$.