

SMALL, SIMPLE NUMBER SYSTEMS

We are all familiar with the notions “an even plus an even is even,” “an even plus an odd is odd,” “an odd times an odd is odd,” etc... If we use the labels 0 for even and 1 for odd, we get the following addition and multiplication tables.

+	0	1
0	0	1
1	1	0

×	0	1
0	0	0
1	0	1

When we were looking for possible solutions to $X^3 + 2X + 1 \equiv 0 \pmod{5}$ we were using the following addition and multiplication tables.

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

×	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

What sort of table do we get if we work modulo 6 instead of 5?

What do we see in these tables?

What will we see in all such tables?

Which of our axioms for integers will these and similar number systems satisfy?