Math 621 Spring 2013

Week 14: Dedekind Domains

Major Definitions: dimension of a ring, irreducible ideal, Dedekind domain, fractional ideal

Major Theorems: a theorem about Dedekind domains

Exercises:

1 – 5. Do exercises 6, 7, 9, 10 and 11 on page 408. Note on #7: $S^{-1}R$ can also be the quotient field of $R$, but that’s a Dedekind domain by Hungerford’s definition.

6. a) Let $R$ be a local ring with maximal ideal $M$ and let $K = R/M$. Show that the $R$-module $M/M^2$ is a vector space over $K$, and that if $R$ is Noetherian, this is a finite dimensional vector space.

b) Suppose further that $R$ is Noetherian of dimension 1. Show that $R$ is a discrete valuation ring if and only if $\dim_K (M/M^2) = 1$. 
