Math 621 Spring 2013
Week 8: Products of Rings

Major Definitions: Direct product and direct sum

Major Theorems: Chinese Remainder Theorem

Exercises: (remember our conventions on rings and modules)

1. Let $R_1, R_2, \ldots, R_n$ be rings. Show that the ideals of the finite direct product ($= \text{direct sum}$) $\prod_{i=1}^{n} R_i$ are of the form $\prod_{i=1}^{n} I_i$, where $I_i$ is an ideal of $R_i$ for each $i$.

2. Let $R$ be a ring. For the moment let the endomorphisms of $R$ act on the right, so that if $f, g \in \text{End}(R)$, then $fg$ acts on $R$ by $rfg$ ($f$ first, then $g$). This is how we let $n \times n$ matrices act on row-vectors in $\mathbb{R}^n$, for example. Using this convention, prove that $\text{End}(R)$ and $R$ are isomorphic as left $R$-modules.

3. Let $m$ and $n_1, n_2, \ldots, n_m$ be positive integers. Let $S_1, \ldots, S_m$ be simple pair-wise non-isomorphic $R$-modules. Prove that

$$\text{End}\left(\bigoplus_{i=1}^{m} \left(\bigoplus_{j=1}^{n_i} S_i\right)\right) \cong \bigoplus_{i=1}^{m} \text{End}\left(\bigoplus_{j=1}^{n_i} S_i\right).$$