Exercise 18. For all $s = \sigma + it$ with $0 < \sigma < 1$, show that

$$\int_0^\infty u^{s-1}e^{-iu} \, du = e^{-i\pi s/2}\Gamma(s).$$

Hint: integrate $u^{s-1}e^{-u}$ along the quarter circle contour of radius $R$ in the first quadrant and let $R \to \infty$.

Exercise 19. Using exercise 18, show that

$$\int_0^\infty u^{\sigma-1}\sin u \, du = \Gamma(\sigma)\sin(\pi \sigma/2)$$

for all $0 < \sigma < 1$. Then let $\sigma \to 0^+$ to find $\int_0^\infty (\sin u/u) \, du$.

Exercise 20: Show that

$$-\int_B^\infty \frac{\sin u}{u} \, du \ll \min\{1, 1/B\}$$

for all $B > 0$, where the implicit constant is absolute. Hint: integrate by parts for the case where $B \geq 1$. 