6.1 Exercises

1. (a) What is a one-to-one function?
(b) How can you tell from the graph of a function whether it is one-to-one?

2. (a) Suppose $f$ is a one-to-one function with domain $A$ and range $B$. How is the inverse function $f^{-1}$ defined? What is the domain of $f^{-1}$? What is the range of $f^{-1}$?
(b) If you are given a formula for $f$, how do you find a formula for $f^{-1}$?
(c) If you are given the graph of $f$, how do you find the graph of $f^{-1}$?

3–16 A function is given by a table of values, a graph, a formula, or a verbal description. Determine whether it is one-to-one.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>1.5</td>
<td>2.0</td>
<td>3.6</td>
<td>5.3</td>
<td>2.8</td>
<td>2.0</td>
</tr>
</tbody>
</table>

$y$

4. $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>1.0</td>
<td>1.9</td>
<td>2.8</td>
<td>3.5</td>
<td>3.1</td>
<td>2.9</td>
</tr>
</tbody>
</table>

5. $y$

6. $x$

7. $f(x) = x^2 - 2x$

8. $f(x) = 10 - 3x$

9. $g(x) = 1/x$

10. $g(x) = |x|$

11. $h(x) = 1 + \cos x$

12. $h(x) = 1 + \cos x$, $0 \leq x \leq \pi$

13. $f(t)$ is the height of a football $t$ seconds after kickoff.

14. $f(t)$ is your height at age $t$.

15. Assume that $f$ is a one-to-one function.
(a) If $f(6) = 17$, what is $f^{-1}(17)$?
(b) If $f^{-1}(3) = 2$, what is $f(2)$?

16. If $f(x) = x^2 + x^3 + x$, find $f^{-1}(3)$ and $f(f^{-1}(2))$.

17. If $h(x) = x + \sqrt{x}$, find $h^{-1}(6)$.

20. The graph of $f$ is given.
(a) Why is $f$ one-to-one?
(b) What are the domain and range of $f^{-1}$?
(c) What is the value of $f^{-1}(2)$?
(d) Estimate the value of $f^{-1}(0)$.

21. The formula $C = \frac{5}{9}(F - 32)$, where $F \geq -459.67$, expresses the Celsius temperature $C$ as a function of the Fahrenheit temperature $F$. Find a formula for the inverse function and interpret it. What is the domain of the inverse function?

22. In the theory of relativity, the mass of a particle with speed $v$ is

$$m = f(v) = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where $m_0$ is the rest mass of the particle and $c$ is the speed of light in a vacuum. Find the inverse function of $f$ and explain its meaning.

23–28 Find a formula for the inverse of the function.

23. $f(x) = 3 - 2x$

24. $f(x) = \frac{4x - 1}{2x + 3}$

25. $f(x) = 1 + \sqrt{x + 3x}$

26. $y = x^2 - x$, $x \geq \frac{1}{2}$

27. $y = \frac{1 - x}{1 + \sqrt{x}}$

28. $f(x) = 2x^2 - 8x$, $x \geq 2$

29–30 Find an explicit formula for $f^{-1}$ and use it to graph $f^{-1}, f$, and the line $y = x$ on the same screen. To check your work, see whether the graphs of $f$ and $f^{-1}$ are reflections about the line.

29. $f(x) = x^4 + 1$, $x \geq 0$

30. $f(x) = \sqrt{x^2 + 2x}$, $x > 0$

31–32 Use the given graph of $f$ to sketch the graph of $f^{-1}$.

31. $y$

32. $y$