1. Estimate the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ with an error less than $10^{-6}$.

This is an alternating series, so a given partial sum $s_n$ is within the next summand $a_{n+1}$ of the sum of the series. Since $10!$ is larger than $10^6$, we know that

$$\frac{1}{1} + \frac{1}{2} - \frac{1}{6} + \cdots - \frac{1}{9!}$$

is within $\frac{1}{10!} < 10^{-6}$ of the sum of the series.

2. Use the sum of the first five terms to approximate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{3^n + 5^n}$. Estimate the error.

The sum of the first five terms is

$$\frac{1}{3+5} + \frac{1}{9+25} + \frac{1}{27+125} + \frac{1}{81+625} + \frac{1}{243+3125}.$$

One handy way to estimate the error is to note that the terms here are always less than $\frac{1}{5^n}$ and $\sum 5^{-n}$ is a geometric series. Thus, the error is less than

$$\frac{1}{5^6} + \frac{1}{5^7} + \frac{1}{5^8} + \frac{1}{5^9} + \cdots = \frac{\frac{1}{5^6}}{1 - (1/5)} = \frac{1}{4 \cdot 5^5}.$$

3. Suppose $\sum a_n$ is an infinite series. What can you say about the series in the following cases?

a) $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = 2$.

b) $\lim_{n \to \infty} \sqrt[n]{|a_n|} = 1/2$.

c) The series alternates and $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$.

For (a), the series diverges by the ratio test. For (b), the series converges absolutely by the root test. For (c), nothing can be said definitively. For example, the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$, $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ and $\sum_{n=1}^{\infty} (-1)^{n-1}n$ all fit the hypotheses, but the first converges absolutely, the second converges conditionally, and the last diverges.