Exercises from section 1.2

1. By using the Euclidean algorithm, find the greatest common divisor of
   (a) 7469 and 2464; (b) 2689 and 4001;
   (c) 2947 and 3997; (d) 1109 and 4999.

2. Find the greatest common divisor $g$ of the numbers 1819 and 3587, and then find integers $x$ and $y$ to satisfy

   $1819x + 3587y = g$.

6. Prove that the product of three consecutive integers is divisible by 6, of four consecutive integers by 24.

11. Prove that $4 \nmid (n^2 + 2)$ for any integer $n$.

14. Prove that if $n$ is odd, $n^2 - 1$ is divisible by 8.

24. Prove that no integers $x, y$ exist satisfying $x + y = 100$ and $(x, y) = 3$.

25. Prove that there are infinitely many pairs of integers $x, y$ satisfying $x + y = 100$ and $(x, y) = 5$.

44. Prove that every positive integer is uniquely expressible in the form

   $2^{j_0} + 2^{j_1} + 2^{j_2} + \cdots + 2^{j_m}$

   where $m \geq 0$ and $0 \leq j_0 < j_1 < j_2 < \cdots < j_m$.

Exercises from section 1.3

10. Prove that any positive integer of the form $3k + 2$ has a prime factor of the same form; similarly for each of the forms $4k + 3$ and $6k + 5$. 
31. Prove that no polynomial $f(x)$ of degree $> 1$ with integral coefficients can represent a prime for every positive integer $x$.

The following “hint” is provided in the back of the book for this exercise. If $f(j) = p$, then $f(j + kp) - f(j)$ is a multiple of $p$ for every $k$, so $f(j + kp)$ has the same property. (Note that you should prove these assertions if you choose to use them in your solution.)