1. Find the following absolute values: $|246|_2$, $|246|_3$, $|246|_5$, $|-4879|_{17}$, $|34/55|_{11}$.

2. Recall from calculus that we say an infinite series $\sum_{n=1}^{\infty} a_n$ converges if the sequence of partial sums $\{s_n\}$, where

$$s_n = a_1 + \cdots + a_n$$

converges. Of course, we’ve seen that “converges” depends on the absolute value one is using. In freshman calculus, that’s the usual absolute value (and the $a_n$’s are real numbers). An elementary result is that the terms of the series must tend to zero if the series converges. One typical mistake students make, though, is to claim that the series $\sum a_n$ converges when the terms $a_n \to 0$ (i.e., $|a_n| \to 0$). This is false, as the example $a_n = 1/n$ shows.

Now let’s try this with $p$-adic numbers. Suppose $p$ is a prime and $\{a_n\} \subset \mathbb{Q}_p$ is a sequence of $p$-adic numbers. Once again, we say the infinite series $\sum a_n$ converges if the sequence of partial sums $\{s_n\}$ converges (using the $p$-adic absolute value). Prove that, in contrast to the case with real numbers, the infinite series $\sum a_n$ converges if and only if $|a_n|_p \to 0$ as $n \to \infty$. (HINT: recall that $\mathbb{Q}_p$ is topologically complete, i.e. all Cauchy sequences converge, and that the $p$-adic absolute value satisfies something stronger than the usual triangle inequality.)