

## MATH 211 Skills Review: (Implicit) domains of functions

If a function  $f$  is described by giving a formula for  $f(x)$  in terms of  $x$  (writing, e.g., “Let  $f(x) = \frac{x^2-3}{x+1}$ ,” or “Define  $f$  by  $f(x) = \sqrt{2x-5}$ ”) without *explicitly* stating the intended domain, then the domain of  $f$  is meant to be the domain of the variable  $x$  in the expression giving  $f(x)$ , that is, the set of all real numbers at which that expression is defined. This set is often called the *implicit domain* of the function: It is *implicit* in our not having stated the domain *explicitly* that this is the intended domain.] Thus, for instance, if  $f$  is given by  $f(x) = \frac{x^2-3}{x+1}$ , then the implicit domain of  $f$  is  $\{x : x \neq -1\}$ , the set of all real numbers excluding  $-1$ , since the expression  $\frac{x^2-3}{x+1}$  is defined at a number  $c$  exactly if  $c \neq -1$ . Similarly, if  $f$  is given by  $f(x) = \sqrt{2x-5}$ , then the implicit domain of  $f$  is the set  $\{x : x \geq 5/2\}$ , that is, the interval  $[5/2, \infty)$ , since the expression  $\sqrt{2x-5}$  is defined at a number  $c$  exactly if  $2c-5 \geq 0$ , i.e. if  $c \geq 5/2$ .

Until we begin working with exponential functions and logarithmic functions, most of the functions we deal with will be given by expressions that are *algebraic* in the variable, that is, that are built up from the variable and constants using addition, subtraction, negation, multiplication, division, integer powers, and the the extraction of roots. The domain of the variable in such expressions is always the set of all real numbers that do not, when substituted for the variable, produce negative values under even radicals (i.e.  $\sqrt{\quad}$ ,  $\sqrt[4]{\quad}$ ,  $\sqrt[6]{\quad}$ , and so on) or a value of 0 in the denominator of a fraction. The two examples given above illustrate each of these sorts of restriction on the domain. Keep in mind that

- a rational power may conceal an even radical: E.g.  $x^{3/4}$  is, by definition, equivalent to  $(\sqrt[4]{x})^3$ , so the domain of  $x$  in the expression  $x^{3/4}$  is  $[0, \infty)$ , and that of  $s$  in  $(9-7s)^{3/4}$  is  $\{s : 9-7s \geq 0\}$ , i.e.  $\{s : s \leq 9/7\}$ , i.e.  $(-\infty, 9/7]$ ;
- negative powers conceal fractions: E.g.  $x^{-2}$  is, by definition, equivalent to  $\frac{1}{x^2}$ , so the domain of  $x$  in the expression  $x^{-2}$  is all real numbers excluding zero, and the domain of  $s$  in the expression  $(3s-5)^{-2}$  is the set of all real numbers excluding  $5/3$  (since the value of  $3s-5$  at  $5/3$  is 0).

As you can see from the examples just given, you may need to brush up on your skills at solving linear inequalities and polynomial equations to find domains of the sorts of algebraic functions given in the exercises below.

### EXERCISES:

Find the implicit domain of each function listed below.

$$f(x) = \frac{2}{x-5}$$

$$f(x) = \sqrt{7-6x}$$

$$f(x) = \frac{x}{4-3x}$$

$$f(x) = x\sqrt{x-3}$$

$$f(x) = \frac{2x}{x-5} + \frac{3}{x}$$

$$f(x) = \frac{4}{\sqrt{3x-4}}$$

$$f(x) = \frac{x-5}{x^2-4}$$

$$f(x) = \frac{4x^2+x}{\sqrt[4]{3x+7}}$$

$$f(x) = \frac{2x-5}{x^2-3x-18}$$

$$f(x) = \sqrt[3]{2x+1}$$

$$f(x) = \frac{9-16x}{x^2+4}$$

$$f(x) = \frac{\sqrt{x}-2}{3x^2-5x}$$

$$f(x) = (2x-3)^{-2}$$

$$f(x) = \frac{-5x}{4-\sqrt{x}}$$

$$f(x) = 1+x^{-1}+3x^{-2}$$

$$f(x) = (7-2.3x)^{-5/4}$$

$$f(x) = (2x+5)^{2/3}$$

$$f(x) = (2x+5)^{3/2}$$