MATH 211 Skills Review: Graphs of power functions and absolute value; transformations

<u>**Powers of x:**</u>

1. You should know the graphs of the equations

$$y = x^2, \quad y = x^4 \quad y = x^6, y = x^8, \quad \text{etc.},$$

how they are similar and how they change shape as you increase the exponent.

2. Same as 1 for the equations

$$y = x^3, \quad y = x^5, \quad y = x^7, y = x^9 \quad \text{etc.}$$

3. Same as 1 for the equations

$$y = x^{-2}, \quad y = x^{-4}, \quad y = x^{-6}, \quad y = x^{-8}, \quad \text{etc.},$$

that is, for

$$y = \frac{1}{x^2}, \quad y = \frac{1}{x^4}, \quad y = \frac{1}{x^6}, \quad y = \frac{1}{x^8}, \quad \text{etc.}$$

4. Same as 1 for the equations

$$y = x^{-3}, \quad y = x^{-5}, \quad y = x^{-7}, \quad y = x^{-9}, \quad \text{etc.},$$

that is, for

$$y = \frac{1}{x^3}, \quad y = \frac{1}{x^5}, \quad y = \frac{1}{x^7}, \quad y = \frac{1}{x^9}, \quad \text{etc.}$$

5. Same as 1 for the equations

$$y = x^{1/2}, \quad y = x^{1/4}, \quad y = x^{1/6}, \quad y = x^{1/8}, \quad \text{etc.},$$

that is, for

$$y = \sqrt{x}, \quad y = \sqrt[4]{x}, \quad y = \sqrt[6]{x}, \quad y = \sqrt[8]{x}, \quad \text{etc.}$$

You should know that these are the upper halves of the graphs of

$$x = y^2, \quad x = y^4, \quad x = y^6, \quad x = y^8, \quad \text{etc.}$$

(and why *only* the upper halves).

6. Same as 1 for the equations

$$y = x^{1/3}, \quad y = x^{1/5}, \quad y = x^{1/7}, \quad y = x^{1/9}, \quad \text{etc.},$$

that is, for

$$y = \sqrt[3]{x}, \quad y = \sqrt[5]{x}, \quad y = \sqrt[7]{x}, \quad y = \sqrt[9]{x}, \quad \text{etc.}$$

You should know that these are the SAME as the graphs of

$$x = y^3, \quad x = y^5, \quad x = y^7, \quad x = y^9, \quad \text{etc.}.$$

These graphs are available for viewing or printing at

`www.math.niu.edu/~kholland/211/ALGREV`

under the six titles beginning Figures for .... Spend some time studying and comparing them so that you can reconstruct them on demand.

Absolute value:

You should know the graph of $y = |x|$. (You may find it in your Bittinger text.) Note that it can be obtained from the graph of $y = x$ by reflecting across the $x-$axis that portion of the graph that lies below the $x-$axis. (Do this now to get the idea fixed in your mind.) More generally, for any function f, the graph of $y = |f(x)|$ can be obtained from the graph of $y = f(x)$ in the same manner — by reflecting across the $x-$axis that portion of the graph that lies below the $x-$axis.

Transformations of known graphs:

You should be able, given the graph of a function f, and POSITIVE constants k and m, to obtain the graph of

- $y = kf(x)$ by a vertical stretch or compression (depending on whether $k > 1$ or $k < 1$, respectively);
- $y = f(kx)$ by a horizontal stretch or compression (depending on whether $k < 1$ or $k > 1$, respectively);
- $y = f(x) + k$ and $y = f(x) - k$ by a vertical shift (up k units or down k units, respectively);
- $y = f(x + k)$ and $y = f(x - k)$ by a horizontal shift (left k units or right k units, respectively);
- $y = -f(x)$ by reflection across the $x-$axis;
- $y = f(-x)$ by reflection across the $y-$axis.

You should be able to combine these techniques to transform the graph of a function f into the graph of, for example, $y = -2f(x-3) + 5$ by first shifting 3 units to the right, then stretching vertically by a factor of 2, then reflecting across the $x-$axis, then shifting up by 5 units. The following exercises, in increasing order of complexity, will help you cement these skills.

EXERCISES:

1. (a) On a single set of axes, graph the three equations $y = x^2$, $y = 3x^2$, $y = \frac{1}{3}x^2$. Plot points at $x = 0$, $x = \pm 1$ and $x = \pm 2$ on all three graphs.

   (b) On a single set of axes, graph the three equations $y = x^2$, $y = (x-3)^2$, $y = (x+3)^2$. Plot the vertex of each graph. How are the vertices related?

   (c) On a single set of axes, graph the three equations $y = x^2$, $y = x^2 - 3$, $y = x^2 + 3$. Plot the vertex of each graph. How are the vertices related?

   (d) On a single set of axes, graph the three equations $y = \sqrt{x}$, $y = \sqrt{-x}$, $y = -\sqrt{x}$. What are the domain and range of the function defined in each case?

   (e) On a single set of axes, graph the three equations $y = \sqrt[3]{x}$, $y = \sqrt[3]{8x}$, $y = \sqrt[3]{\frac{1}{8}x}$. Now on a new set of axes, graph the three equations $y = \sqrt[3]{x}$, $y = 8\sqrt[3]{x}$, $y = \frac{1}{8}\sqrt[3]{x}$.

   (f) Copy the graph of $f(x) = 2^x$ from p.287 of your Bittinger text onto a set of axes. On the same set of axes, graph $y = f(x+3) = 2^{x+3}$, $y = f(x-3) = 2^{x-3}$, $y = f(x) + 3 = 2^x + 3$ and $y = f(x) - 3 = 2^x - 3$.

   (g) As above, copy a graph of $f(x) = 2^x$. On the same set of axes, graph $y = f(-x) = 2^{-x}$ and $y = -f(x) = -2^x$.

2. In the following exercises, graph the list of equations **in the order given**, seeing each graph as an appropriate transformation of the one preceding it. Tell in words what action you have taken in moving from each graph in the series to the next.

   (a) $y = \sqrt{x}$, $y = \sqrt{x+3}$, $y = 2\sqrt{x+3}$, $y = -2\sqrt{x+3}$, $y = 1 - 2\sqrt{x+3}$

(b) $y = \dfrac{1}{x^2}$, $y = \dfrac{3}{x^2}$, $y = \dfrac{3}{(x-3)^2}$, $y = \dfrac{3}{(x-3)^2} + 4$

(c) $y = \sqrt[4]{x}$, $y = \sqrt[4]{x+3}$, $y = \sqrt[4]{3-x}$ $(= \sqrt[4]{(-x)+3})$

(d) $y = |x|$, $y = |x+4|$, $y = |(5x)+4|$, $y = -|5x+4|$

(e) $y = \sqrt[3]{x}$, $y = \sqrt[3]{-x}$, $y = \sqrt[3]{-x-2}$ $(= \sqrt[3]{-(x+2)})$

3. Graph the given function. [Now you will have to go from your "basic" function to the end result by finding intermediate, one action steps as was done for you in exercise 2.]

   (a) $y = -3\sqrt{x+2}$

   (b) $y = 5 - \dfrac{1}{x}$

   (c) $y = \dfrac{1}{(4-x)^3}$

   (d) $y = \sqrt[3]{2x+9}$

   (e) $y = \sqrt{9-4x}$

   (f) $y = 4(x+3)^{-2} - 5$

4. The graphs of the following can be found as transformations of the graph of $y = |x|$. They can also be found by seeing them as $y = |f(x)|$ and reflecting across the $x-$axis those portions of the graph of $y = f(x)$ that lie below the $x-$axis. Try each graph both ways, and see which you find easier.

   (a) $y = |x-3|$

   (b) $y = |2x+5|$

   (c) $y = |4+x|$

   (d) $y = |3-5x|$

   (e) $y = |-x-12|$

5. Graph the following by using an appropriate combination of the techniques practiced so far.

   (a) $y = |x^2 - 2|$ (Hint: Go from $y = x^2$ to $y = x^2 - 2$ to $y = |x^2 - 2|$.)

   (b) $y = |\sqrt{x} - 2|$

   (c) $y = |2 - \sqrt{x}|$ (Your graph should be the same as that in part b. Can you explain why?)

   (d) $y = |\frac{1}{x} + 3|$