MATH 211 Skills Review: Quadratic functions and thier graphs

A quadratic function is one of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. You should be able (easily and quickly!) to do any of the following:

- Solve a quadratic equation $ax^2 + bx + c = 0$ for x by one of the following methods:
  - factorization of $ax^2 + bx + c$ into a product of the form $(Ax - B)(Cx - D)$, giving solutions $x = -B/2A$ and $x = -D/C$;
  - using the quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, which you should *immediately and permanently* memorize, since you will be using it a lot;
  - completing the square.
- Graph any quadratic function $f(x) = ax^2 + bx + c$, finding and labeling at least 5 points on the graph, including
  - its y-intercept $(0, f(0))$, i.e. $(0, c)$,
  - its x-intercepts, if any, which are given by the solutions of the associated equation $ax^2 + bx + c = f(x) = 0$;
  - its vertex, $(-b/2a, f(-b/2a))$.

  It should not take you more than a couple of minutes to accomplish all this for a given function f. It is considered a routine background skill.

You should know, without actually graphing, that the graph of $f(x) = ax^2 + bx + c$ has the same shape and direction as the graph of $y = ax^2$, that it is a parabola opening up if $a > 0$ and down if $a < 0$, that it is symmetric with respect to its axis, the line $x = -b/2a$.

EXERCISES:

1. Solve the following equations. [Note that while one can always use the quadratic formula, if the relevant quadratic happens to factor over the integers, then factoring is generally quicker and less likely to produce arithmetic simplification errors. Whenever you solve by factoring, it is a good habit to mentally multiply the product back out to make certain you have not made a mistake.]

   (a) $3x^2 - 4x = 0$

   (b) $2x + 5x^2 = 0$

   (c) $x^2 - 4 = 0$

   (d) $3x^2 - 5 = 0$

   (e) $x^2 + 4$

   (f) $4x^2 + 9 = 0$

   (g) $x^2 + 2x - 24 = 0$

   (h) $12x^2 + 19x + 5 = 0$

   (i) $24 - 2x - x^2 = 0$

   (j) $x^2 + 2x - 20 = 0$

   (k) $12x^2 + 19x + 30 = 0$

   (l) $6x^2 + x + 5 = 0$

   (m) $-11x^2 + x - 1 = 0$

(n) $9 + 6x + x^2 = 0$

(o) $4x^2 - 44x + 121 = 0$

2. Solve for x. (Except in a)-c), you will want to rewrite the equation in an equivalent form $ax^2 + bx + c = 0$, and proceed as you did in the exercises of part 1 above.

(a) $x^2 = 4$

(b) $3x^2 = 5$

(c) $6x^2 = -5$

(d) $6x^2 = -2x^2$

(e) $5x^2 = 2x$ (careful!)

(f) $x^2 + 2x = 24$

(g) $x^2 + 12x + 3 = 10x + 21$

(h) $x^2 + 2x = 24$

(i) $3x^2 + 2x = 2x^2 + 20$

(j) $4x^2 + \frac{19}{3}x = -10$

(k) $x + 5 = 6x^2$

(l) $x = 1 + 11x^2$

(m) $(x - 5)^2 = 12$

(n) $(x + 2)(x - 3) = 5$

(o) $-x(2x + 10) = x - 1$

3. Graph the following functions, labeling the coordinates of at least 5 points on the graph, including all x- and y-intercepts and the vertex. [You already found the $x-$intercepts of many of these functions in the exercises in part 1 above.]

(a) $f(x) = 3x^2 - 4x$

(b) $f(x) = x^2 - 4$

(c) $f(x) = 4x^2 + 9$

(d) $f(x) = -4x^2 - 121$

(e) $f(x) = x^2 + 2x - 24$

(f) $f(x) = x^2 + 2x - 20$

(g) $f(x) = 6x^2 + x + 5$

(h) $f(x) = x^2 + 6x + 9$

(i) $f(x) = -4x^2 + 44x - 121$

(j) $f(x) = (x - 5)^2 - 12$

(k) $f(x) = -x(x + 2) + 3x^2 - 4x(3x + 1)$