Practice problems on limits (WITHOUT L'HOPITAL'S RULE!!)

1. $\lim_{x\to 5} \dfrac{\sin^2(x-5)}{x-5}$
2. $\lim_{x\to 1} \dfrac{1-\cos(\pi x)}{x-1}$
3. $\lim_{x\to \infty} \dfrac{x^{3/2}-4x+1}{-3x^2-5x}$
4. $\lim_{x\to \infty} \dfrac{x^{5/2}-4x+1/x}{-3x^2-5x}$
5. $\lim_{x\to 0} \dfrac{x^{5/2}-4x+1/x}{-3x^2-5/x^2}$
6. $\lim_{x\to \infty} \dfrac{\sqrt{5x^4+3x+1}}{x^3-2x}$
7. $\lim_{x\to 6} \dfrac{\sqrt{x+3}-\sqrt{x^2-27}}{x^2-36}$
8. $\lim_{x\to 1} \dfrac{x^4-1}{3x^2+x-4}$
9. $\lim_{x\to 0^+} \left(1/x-2/x^2\right)$
10. $\lim_{x\to 0} \dfrac{x\sin x+\tan x}{x^2}$
11. $\lim_{x\to 0} \dfrac{\tan^{-1}(x)}{x}$ (Hint: Set $u=\tan^{-1}(x)$.)
12. $\lim_{x\to 0} \dfrac{x}{\sqrt{1-\cos x}}$
13. $\lim_{x\to \infty} \theta^2\tan(1/\theta)$
14. $\lim_{x\to \infty} \dfrac{\pi/2-\tan^{-1}x}{1-1/x}$
15. $\lim_{x\to 0^+} \dfrac{1}{\sin x}$
16. $\lim_{x\to 0^-} \dfrac{1}{\sin x}$
17. $\lim_{x\to 0} \dfrac{1}{\sin x}$
18. $\lim_{x\to 0^+} \dfrac{1}{\sin x-x}$ (Careful!)
19. $\lim_{x\to 0^-} \dfrac{1}{\sin x-x}$
20. $\lim_{x\to 0^+} \dfrac{\sin^3 x-x^4}{\tan^2 x+x^3}$

SOME BASIC LIMITS: (most easily remembered with graphs!)

- For $a > 1$, $\lim_{x\to\infty} a^x = \infty$ and $\lim_{x\to-\infty} a^x = 0^+$.
- For $0 < a < 1$, $\lim_{x\to\infty} a^x = 0^+$ and $\lim_{x\to-\infty} a^x = \infty$.
- $\lim_{x\to\infty} \ln(x) = \infty$ and $\lim_{x\to 0^+} \ln(x) = -\infty$.

Use these basic limits, together with appropriate substitutions and the laws of logarithms, if necessary, to compute the following limits:

1. The limits at ∞ of
$$e^{-3x+2},\ \ e^{3x-2},\ \ 2^{-x^2+1},\ \ 2^{1/x},\ \ 2^{-1/x},\ \ (\sqrt{3}/2)^{\sqrt{x}},\ \ (.3)^{1/x},\ \ (2/3)^{-3x+5},\ \ (1-1/e)^{x^3}.$$
2. The limits at $-\infty$ of the same functions.
3. $\lim_{x\to\infty} \ln(x^2 + 1/x)$
4. $\lim_{x\to\infty} (\ln(x) - \ln(x^2+1)$
5. $\lim_{x\to 0^+} (\ln(\sin x)) - \ln(x))$
6. $\lim_{x\to 0^+} e^{\frac{\sin\sqrt{x}}{\sqrt{x}}}$
7. $\lim_{x\to-\infty} \sqrt{e^x}$
8. $\lim_{x\to\infty} \dfrac{5e^{3x} - e^{2x} + 3}{-2e^{2x} + 5}$
9. $\lim_{x\to-\infty} \dfrac{5e^{3x} - e^{2x} + 3}{-2e^{2x} + 5}$
10. $\lim_{x\to\infty} \dfrac{2e^{-3x} + 5e^x - 4}{e^x - 7}$
11. $\lim_{x\to 0} \dfrac{\sqrt{e^x} - 1}{e^x - 1}$