

## Math 230, Sec.6, Spring 08, Exam 3

1. (SHORT ANSWER, NO PARTIAL CREDIT, NO WORK NEED BE SHOWN)

(a)  $\lim_{n \rightarrow \infty} \sqrt[n]{n}$

(b)  $\lim_{n \rightarrow \infty} \sqrt[n]{n!}$

(c)  $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{5x^4 + 3x^2 + 2}}$

(d)  $\lim_{n \rightarrow \infty} \frac{3^n}{n^2 4^n}$

(e)  $\lim_{n \rightarrow \infty} \frac{100^n}{n!}$

(f)  $\lim_{n \rightarrow \infty} \frac{(\ln(n))^{12}}{n^{1.1}}$

(g)  $\lim_{n \rightarrow \infty} \frac{\ln(n+1)}{\ln(n)}$

2. (SHORT ANSWER, NO PARTIAL CREDIT, NO WORK NEED BE SHOWN) In each case, answer CONVERGES, DIVERGES or INCONCLUSIVE.

(a) If  $a_n$  are nonnegative, decreasing to 0, then  $\sum a_n \dots$

(b) If  $0 \leq a_n \leq \frac{1}{n}$  eventually, then  $\sum a_n \dots$

(c) If  $\frac{1}{n^3} \leq a_n$  eventually, then  $\sum a_n \dots$

(d) If  $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$  then  $\sum a_n \dots$

3. (SHORT ANSWER, NO PARTIAL CREDIT, NO WORK NEED BE SHOWN) State whether the following series CONVERGE or DIVERGE.

(a)  $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

(b)  $\sum_{n=2}^{\infty} \frac{\ln(n)}{n}$

(c)  $\sum_{n=0}^{\infty} \frac{n!}{100^n}$

(d)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + 1}$

(e)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2 + 1}}$

(f)  $\sum_{n=1}^{\infty} \cos(n\pi)$

4. Sum the series  $\sum_{n=3}^{\infty} \frac{4 + (-3)^n}{5^{n+1}}$  or show that it diverges. Display your work carefully and completely.

5. Compute  $\int_2^{\infty} \frac{2}{x \ln^3 x} dx$ . Display your work carefully and completely.

6. Determine whether the following integrals converge or diverge. Justify your answers carefully, stating any tests that you use.

(a)  $\int_2^{\infty} \frac{n^{3/2}}{n^3 + 1} dx$

(b)  $\int_1^{\infty} \sec^{-1} x dx$

7. Determine whether each series DIVERGES, CONVERGES ABSOLUTELY, or CONVERGES CONDITIONALLY. Support your answer with a clear argument based on one or more stated tests.

(a)  $\sum \frac{n^2(-2)^n}{n!}$

(b)  $\sum \frac{(-1)^n}{3n+1}$

(c)  $\sum \left(\frac{-3}{n}\right)^n$