014 – Comparison of Heart Rate and Body Mass in Animals

Prerequisites: Proportionality/Geometric Similarity
Regression

It is well-known that small warm-blooded animals have much higher resting pulse rates than large ones. For examples, a canary, with a body mass of 20 grams, may have a heart rate of 1000 beats/minute, as compared to 70-80 beats/minute for a human being. In this project, we build a model relating heart rate to body mass, and then test the validity of the model using actual data.

Warm-blooded animals at rest use large quantities of energy to maintain body temperature, due to heat loss through the body surface. Let us make the following assumptions about energy balance over a fixed time interval (1 minute, say):

A. Energy lost is proportional to the surface area of the body.

B. Energy gained is proportional to the amount of blood flowing through the lungs, the source of oxygen for the body.

C. Energy lost = energy gained for a body in temperature equilibrium.

Let m = mass, r = pulse rate, S = surface area, H = volume of blood pumped by the heart in one stroke.

Problem 1. Using assumptions A, B, C and any assumptions about geometric similarity of animals which you wish to make, develop a <u>theoretical</u> model of proportionality between r and m, for a warm-blooded animal <u>at rest.</u> Your conclusion should be of the form $r \propto m^d$, where d is a constant. Carefully justify your numerical choice for d.

Here are values of m and r for selected mammals and birds <u>at rest:</u>

	Mass (grams)	pulse rate (beats/minute)
Birds		
canary	20	1000
pigeon	300	185
crow	341	378
buzzard	658	300
wild duck	1100	190
hen	2000	312
domestic duck	2300	240
turkey	8750	193
Mammals		
mouse	25	670
rat	200	420
guinea pig	300	300
rabbit	2000	205
small dog	5000	120
large dog	30000	85
man	70000	72
horse	450000	38

Problem 2. Carry out the following procedure, using your regression program, for

(a) birds alone; (b) mammals alone; and (c) birds and mammals together.

(i) record and print the data, and plot pulse as a function of mass.

(ii) transform the data by taking logs; print and plot the new values.

(iii) use regression to get the best linear fit for the log values. Plot the residuals vs. ln(mass) to see if they are positive and negative in some random way.

(iv) exponentiate the best linear fit to predict pulse as a function of mass.

(v) Print mass, actual pulse, and predicted pulse to see how successful your equation in (iv) is. Plot the actual and predicted graphs on the same set of axes.

Problem 3. (a) Which is a better fit: the equation in 2(a) or the equation in 2(b)? Give reasons to support your answer.

(b) Are the equations obtained in 2(a) and 2(b) close enough that we can reasonably combine birds and mammals in the same population? Give evidence to support your answer.

Problem 4. How does the theoretical prediction in Problem 1 compare with the best-fit equation in Problem 2(c)? If the agreement is not good, how could the theoretical model be adjusted to produce better agreement?

Now consider the situation when an animal is running or flying at a constant velocity v. In this case, the animal is losing energy in an additional way: it does work to produce kinetic energy $(= \frac{1}{2}mv^2)$. Under these conditions, the pulse rate r will increase from the resting value. The parameter $H =$ (volume of blood pumped by the heart in one stroke) will also increase from the resting value.

Problem 5. Develop a theoretical equation expressing the product $r \cdot H$ in terms of m and v. Your expression will contain some arbitrary constants. Note that $r \cdot H$ is the volume of blood pumped by the heart in one minute.

Here is some data for healthy human beings at various levels of activity:

	150 lb man		120 lb woman		250 lb man	
Activity	r	H	r	H	r	H
rest (0 mph)	68	82	74	64	62	134
walking (3 mph)	91	108	97	86	76	179
jogging (5 mph)	140	127	153	105	117	221

Problem 6. (a) Use the data, and your regression program, to obtain numerical values for the constants in your equation in Problem 5.

(b) Using the equation obtained in (a), calculate the value of $r \cdot H$ for a 120 pound athlete running the marathon (26 miles 385 yards) in 2 hours 8 minutes, the unofficial world record.

(c) Suppose the same 120 pound athlete could sustain an $r \cdot H$ value of 45,000 cm^3/min. How long would this athlete take to run 1 mile?