Non-commutative extensions of Wiener's Lemma in frame and time-frequency analysis

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Outline

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  - $T(\gamma)I = I$ for all $\gamma \in \mathbb{R}^d$;
  - $T(\gamma)(AB) = (T(\gamma)A)(T(\gamma)B)$ for all $\gamma \in \mathbb{R}^d$, $A, B \in \mathcal{B}$.

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- $A \in AP^T(\mathcal{B})$ if the function $\hat{A} : \mathbb{R}^d \to \mathcal{B}$, $\hat{A}(\gamma) = T(\gamma)A$, is continuous (in the topology of $\mathcal{B}$) and Bohr almost periodic.

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$$A_j = \lim_{N \to \infty} \frac{1}{(2N)^d} \int_{[-N,N]^d} e^{-2\pi i \langle \gamma, j \rangle} T(-\gamma) A d\gamma.$$

Notation - continued

- A *weight* is a function $\nu : \mathbb{R}^d \to [1, \infty)$ such that $\nu(t+s) \leq \nu(t)\nu(s),\ t, s \in \mathbb{R}^d$.

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Definition

A *Baskakov algebra* $AP_\nu^T(\mathcal{B})$ is a subalgebra of $AP^T(\mathcal{B})$ containing all elements with the Fourier series summable with weight ν.

$$\|A\|_\nu = \sum_{j\in\mathbb{R}^d} \nu(j)\,\|A_j\| < \infty.$$

Almost periodic noncommutative Wiener's Lemmae

Theorem (R. Balan, IK)

Let ν be an admissible weight. Then the subalgebra $AP^T_\nu(\mathcal{B}) \subset \mathcal{B}$ is inverse closed, that is, if $A \in AP^T_\nu(\mathcal{B})$ is invertible in $\mathcal{B}$ then $A^{-1} \in AP^T_\nu(\mathcal{B})$.

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Let $\nu_\rho(j) = e^{\rho|j|}$, $j \in \mathbb{R}^d$, and $A \in AP_{\nu_\rho}^T(\mathcal{B})$ be invertible in $\mathcal{B}$. Then there exists $\bar\rho > 0$ such that $A^{-1} \in AP_{\nu_{\bar\rho}}^T(\mathcal{B})$.

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Theorem (R. Balan, IK)

For every $A \in AP_{\nu_\rho}^T(\mathcal{B})$ and $\varepsilon > 0$ there exists $\bar\rho > 0$ such that $(\lambda I - A)^{-1} \in AP_{\nu_{\bar\rho}}^T(\mathcal{B})$ for every $\lambda \in \mathbb{C}$ such that $dist(\lambda, \sigma(A)) \geq \varepsilon$.

Immediate Corollaries

Corollary

Assume that $A \in AP_{\nu}^{T}(\mathcal{B})$ *has a (Moore-Penrose) pseudoinverse* $A^{\sharp} \in \mathcal{B}$.

- *If* ν *is admissible then* $A^{\sharp} \in AP_{\nu}^{T}(\mathcal{B})$.
- *If* $\nu = \nu_{\rho}$ *is an exponential weight, then there exists* $\bar{\rho} > 0$ *such that* $A^{\sharp} \in AP_{\nu_{\bar{\rho}}}^{T}(\mathcal{B})$.

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Corollary

Assume that $\mathcal{B} = End\,\ell^p$ *for some* $p \in [1, \infty]$ *and* $A \in \mathcal{B}$ *is invertible. Assume also that the matrix of* A *has summable diagonals. Then this matrix defines invertible operators in* $End\,\ell^q$ *for all* $q \in [1, \infty]$.

Related results

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Assume that $A \in AP^T(\mathcal{B})$ has finitely rationally independent Bohr spectrum. Then the spectrum $\sigma(A)$ in the Banach algebra $\mathcal{B}$ is invariant under rotations around the origin in $\mathbb{C}$.

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Theorem (IK, T. Strohmer)

Let $A \in AP^T_\nu(\mathcal{B})$ *be positive definite and invertible. Then* A *admits a spectral factorization* $A = L^*L$ *such that* $L, L^* \in AP^T_\nu(\mathcal{B})$ *and the Bohr spectra of* L *and* L^{-1} *are non-negative.*

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Corollary (IK, K. Okoudjou (Gabor frames, $T = T_M$))

If a Gabor frame generator belongs to a Wiener Amalgam space then the canonical dual generator does as well.

Localized Frames and Sampling

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Theorem (A. Aldroubi, A. Baskakov, IK)

A set of sampling for some $p \in [1,\infty]$ remains a set of sampling for all $q \in [1,\infty]$ if the sampling operator is sufficiently localized.

Time-Frequency Shifts and HRT

Let $\mathfrak{U} = AP^T(EndL^2(\mathbb{R}^d))$ and $\mathfrak{U}_\nu = AP^T_\nu(EndL^2(\mathbb{R}^d))$, where $T = T_W$.

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Proposition

Let $A \in End(L^2(\mathbb{R}))$ *be such that*

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Then $\sigma(A)$ *has no isolated eigen-values.*

Causal Factorization of ΨDO

ΨDO:

$$(Ax)(t) = \int_{\mathbb{R}^2} \sigma\left(\frac{s+t}{2}, \xi\right) e^{2\pi i (t-s)\cdot\xi} x(s)\, ds\, d\xi;$$

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Theorem (IK, T. Strohmer)

If a ΨDO A in the Sjöstrand's class $\mathfrak{S}$ satisfies $\|A - I\|_{\mathfrak{S}} < 1$, then A admits canonical causal factorization within $\mathfrak{S}$. If in addition $A \in AP_1^T$, then it is enough to have $\|A - I\|_{\mathcal{B}} < 1$.

References

ABK A. Aldroubi, A. Baskakov, and I. Krishtal, Slanted matrices, Banach frames, and sampling, submitted (2007).

BK R. Balan and I. Krishtal, An almost periodic noncommutative Wiener's Lemma, submitted (2008).

KO I. Krishtal and K. Okoudjou, Invertibility of the Gabor frame operator on the Wiener amalgam space, submitted (2007).

The papers are available via http://www.math.niu.edu/~krishtal/ or from ArXiV.