PART 1: TRUE or FALSE

Circle TRUE or FALSE, whichever is correct. (2 points each)

1. If \( \frac{6x}{6} \geq -12 \), then \( x \leq -2 \).  \( \text{TRUE} \) \( \text{FALSE} \)

2. When solving a system of two equations algebraically and you get \( 0 = 0 \), it means that the two lines are the same.  \( \text{TRUE} \) \( \text{FALSE} \)

3. The graph of the inequality \( x + 2y > 10 \) will have a dashed boundary line.  \( \text{TRUE} \) \( \text{FALSE} \)

4. \( y \) varies inversely as \( x \) if there exists a real number \( k \) such that \( y = kx \).  \( \text{TRUE} \) \( \text{FALSE} \)

5. The relation \( \{(-2, 8), (0, 5), (5, 0), (-7, -1)\} \) represents a one-to-one function.  \( \text{TRUE} \) \( \text{FALSE} \)

6. If the discriminant is positive, then the graph of the quadratic function has two \( x \)-intercepts. \( \text{TRUE} \) \( \text{FALSE} \)

7. \( i = -1 \)  \( \text{TRUE} \) \( \text{FALSE} \)

8. \( 2\sqrt{3}x + 5\sqrt{3}x = 21x \)  \( \text{TRUE} \) \( \text{FALSE} \)
PART 2: FILL-IN-THE-BLANK (2 points each)

9. Solve the quadratic equation $ax^2 + bx + c = 0$ for $x$ with $a$, $b$, and $c$, being real numbers, $a \neq 0$.

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Answer

10. Convert the following radical expression to an equivalent expression involving a rational exponent.

\[ \sqrt[3]{2D} \]

Answer \((2D)^{\frac{1}{13}}\)

11. Simplify the expression.

\[ 4\sqrt{x^{20}} \]

Need parentheses!

Answer

12. Given the quadratic equation $x^2 + 8x = -11$,

a. Find the discriminant.

\[ x^2 + 8x + 11 = 0 \]

\[ b^2 - 4ac \]

Answer

b. Determine the number that will complete the square to solve the equation.

\[ (\frac{8}{2})^2 \]

Answer

13. Write the following inequality using interval notation. $x > -9$

Answer \((-9, \infty)\)

14. $p^{2016} = \frac{504}{1216} \cdot \frac{16}{1216}$

\[ i^0 = 1 \]

Answer

2
PART 3: MULTIPLE CHOICE

Circle the correct answer. (2 points each)

\[
\begin{align*}
\begin{cases}
  x - y + z &= -4 \\ 3x + 2y - z &= 5 \\ -2x + 3y - z &= 15
\end{cases}
\end{align*}
\]

15. Solve the system of equations.

a. \((-1, 6, 3)\)  b. \((-1, 5, 2)\)  c. \(\{(x, y, z) | x - y + z = -4\}\)  d. No Solution  e. none of these

16. Graph the solution set of equations.

\[
\begin{align*}
2x - y &\leq -8 \\
 x + 4y &\geq -16
\end{align*}
\]
PART 4: OPEN-ENDED

Show all of your work clearly on this exam for full credit. **You must circle your final answer!!**

17. (4 points) Reduce the following expression to lowest terms.

\[
\frac{(3x + 4)(x - 1)}{(x - 4)(x - 1)} = \frac{3x + 4}{x - 4}
\]

18. (5 points each) Perform the indicated operations and simplify. Leave your answer in factored form.

a. \[
\frac{x^2 + 7x + 12}{6x + 15} + \frac{x^2 - 16}{12x + 30} = \frac{x^2 + 7x + 12}{6x + 15} \cdot \frac{16 + 30}{x^2 - 16} = \frac{(x + 3)(x + 4)}{3(2x + 5)(x - 4)(x + 1)}
\]

b. \[
\frac{4x}{x - 1} + \frac{x + 3}{1 - x} = \frac{4x}{x - 1} - \frac{x + 3}{x - 1} = \frac{4x - (x + 3)}{x - 1} = \frac{4x - x - 3}{x - 1}
\]

\[
\frac{3x - 3}{x - 1} = \frac{3(x - 1)}{x - 1} = 3
\]
19. (5 points) Simplify the following complex fraction. Assume no denominators are 0.

\[
\frac{\frac{1}{x+1}}{\frac{1}{x} - \frac{1}{x}} = \frac{x+1}{x^2-1} = \frac{x+1}{(x-1)(x+1)} = \frac{1}{x-1}
\]

20. (4 points) Write the rational expression with the indicated denominator.

\[
\frac{19x}{2x-6} \times \frac{3}{6x-18} = \frac{19x}{2(x-3)} = \frac{3}{2(x-3)}
\]

21. (6 points) Daniel and Martin are brothers who share a playroom. By himself, Daniel can completely mess up the playroom in 12 minutes, while it would take Martin 20 minutes to do the same thing. How long would it take them to mess up the playroom together?

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Rate (m)</th>
<th>Done in</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daniel</td>
<td>12</td>
<td>(\frac{1}{12})</td>
</tr>
<tr>
<td>Martin</td>
<td>20</td>
<td>(\frac{1}{20})</td>
</tr>
<tr>
<td>Together</td>
<td>(x)</td>
<td>(\frac{1}{x})</td>
</tr>
</tbody>
</table>

Let \(x\) = time together

\[\text{LCM} = 60\]

\[
5x + 3x = 60
\]

\[
x = \frac{60}{8} = 7.5\text{ minutes}
\]
22. (5 points each) Solve the following equations.
   
   a. \[ \frac{1}{2}x + 3 \pm 5 = 7 \
   \]
   
   \[ \frac{1}{2}x + 3 = 2 \quad \text{or} \quad \frac{1}{2}x + 3 = -3 \]
   
   \[ 2 \left( \frac{1}{2}x - 5 \right) = 2 \left( \frac{1}{2}x - 1 \right) \]
   
   \[ x = -10, -2 \]
   
   b. \[ \left( 7 + \frac{7}{x-7} \right) = \left( \frac{x}{x-7} \right) x - 7 \]
   
   \[ 7(x-7) + 7 = x \]
   
   \[ 7x - 49 + 7 = x \]
   
   \[ 7x - 42 = -x \]
   
   \[ 6x - 42 = 0 \]
   
   \[ x = 7 \]
   
   \[ \left[ \text{divide by 0 if you check 7} \right] \]
   
   23. Write each expression in standard \( a + bi \) form.
   
   a. (3 points) \[ (2 + 7i) - (3 - 2i) \]
   
   \[ -1 + 9i \]
   
   b. (5 points) \[ (2 + 7i)(3 - 2i) \]
   
   \[ 6 - 4i + 21i - 14i^2 \]
   
   \[ 6 + 17i + 14 \]
   
   \[ 20 + 17i \]
24. (5 points each) Solve the inequality. Graph the solution set. Write the solution in interval notation.

a. \(|3x - 1| \geq 4\)

\[
\begin{align*}
3x - 1 &\leq -4 \\
\frac{3x - 1 - 4}{-1} &\leq 1 \\
3x &\leq -3 \\
x &\leq -1
\end{align*}
\]

\[
\begin{align*}
3x - 1 &\geq 4 \\
\frac{3x - 1 - 4}{3} &\geq 1 \\
x &\geq 3
\end{align*}
\]

\((-\infty, -1] \cup [3, \infty)\)

b. \(\frac{x + 6}{x - 1} \leq 0\)

\(
\begin{cases}
x - 1 = 0 &\text{or} \\
x + 6 = 0 &\text{or} \\
x = 1 &\text{or} \\
x = -6 &\text{or}
\end{cases}
\)

\((-6, 1)\)

25. (5 points) Find the exact distance between the following pair of points. \((-6, 1)\) and \((3, -2)\)

\[
d = \sqrt{(3 - (-6))^2 + (-2 - 1)^2} = \sqrt{9^2 + 3^2} = \sqrt{90} = 3\sqrt{10}
\]
26. (5 points each) Solve the following systems of equations as an ordered pair.

\[
\begin{align*}
\text{a.} & \quad \begin{cases} 
4x + 3y &= -23 \\
3x - 5y &= 19 
\end{cases} \\
& \quad \begin{cases} 
12x + 9y &= -69 \\
12x + 15y &= -78 
\end{cases} \\
& \quad \begin{cases} 
29y &= -45 \\
\frac{y}{29} &= \frac{9}{29} 
\end{cases} \\
& \quad \begin{cases} 
x &= -2 \\
y &= -5 
\end{cases} \\
\end{align*}
\]

\[(-2, -5)\]

\[
\begin{align*}
\text{b.} & \quad \begin{cases} 
2x + 3y &= 1 \\
y &= x - 3 
\end{cases} \\
& \quad \begin{cases} 
2x + 3(y - 3) &= 1 \\
2x + 3x - 9 &= 1 \\
5x - 9 &= q_+ 
\end{cases} \\
& \quad \begin{cases} 
x &= 2 \\
y &= (2) - 3 = 2 - 3 = -1 
\end{cases} \\
\end{align*}
\]

\[(2, -1)\]

27. (6 points) A movie theater charges $8.00 for adults and $5.00 for children. If there were 40 people altogether and the theater collected $272.00 at the end of the day, how many of them were adults and how many were children? Write a system of equations and use it to answer the question.

\[
\begin{align*}
\text{Let} & \quad a = \text{price of adult ticket} \\
& \quad c = \text{price of children ticket} \\
& \quad 8a + 5c = 272 \\
& \quad 8a + 5c = 272 \\
& \quad 8a + 5c = 272 \\
& \quad 8a + 5c = 272 \\
& \quad 8a + 5c = 272 \\
& \quad 8a + 5c = 272 \\
& \quad 8a + 5c = 272 \\
& \quad 8a + 5c = 272 \\
& \quad 8a + 5c = 272 \\
& \quad \frac{3a}{3} = \frac{72}{3} \\
& \quad a = 24 \\
\end{align*}
\]

There were 24 adults and 16 children.
28. Consider the following pair of functions. \( f(x) = x^2 - 4; \quad g(x) = x + 2 \)

a. (3 points) Find \( g(-3) \).
   \[
   \begin{array}{c}
   g(-3) = (-3) + 2 \\
   = -1
   \end{array}
   \]

b. (4 points) Find \( g \div f \).
   \[
   \begin{array}{c}
   \frac{x+2}{x^2 - 4} \\
   = \frac{x+2}{(x-2)(x+2)} \\
   \end{array}
   \]

29. Simplify the following expressions.

a. (5 points) \( \left(64x^6\right)^{3/2} \)
   \[
   \begin{array}{c}
   \left(\sqrt[3]{64x^6}\right)^3 = (8x^3)^3 \\
   = 512x^9
   \end{array}
   \]

b. (5 points) \( \sqrt[3]{-54y^5 z^{12}} \)
   \[
   \begin{array}{c}
   -3y z^4 \sqrt[3]{2y^2}
   \end{array}
   \]
30. Consider the following polynomial function.

\[ f(x) = -x^2 + 2x + 8 \]

a. (2 points) Decide whether the graph opens up, down, to the left, or to the right.

\[ A = -1 \]

b. (2 points) Write the equation of the axis of symmetry.

\[ x = \frac{-b}{2a} = \frac{-2}{2(1)} = -1 \]

\[ x = 1 \]

c. (2 points) Find the vertex.

\[ f(1) = -(1)^2 + 2(1) + 8 = -1 + 2 + 8 = 9 \]

\[ (1, 9) \]

d. (3 points) Find the intercepts as coordinates.

\[ f(0) = -(0)^2 + 2(0) + 8 = 8 \]

\[ (0, 8) \]

\[ f(-2) = (-2)^2 + 2(-2) + 8 = 4 - 4 + 8 = 8 \]

\[ f(2) = (2)^2 + 2(2) + 8 = 4 + 4 + 8 = 16 \]

\[ (2, 0) \]

\[ (0, 8), (2, 0) \]

e. (2 points) Give the domain.

\[ (-\infty, \infty) \text{ or } \mathbb{R} \]

f. (2 points) Give the range.

\[ (-\infty, 9) \]
g. (5 points) Graph the polynomial function \( f \) using parts a. – f. above. Label, label, label!
31. (5 points each) Solve the following equations.

a. $3x^2 + 2x + 2 = 0$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(3)(2)}}{2(3)} = \frac{-2 \pm \sqrt{-20}}{6} = \frac{-2 \pm 2i\sqrt{5}}{6}$$

$$x = -\frac{1}{3} \pm \frac{\sqrt{5}}{3} i$$

Note: $i$ is on the side

b. $x^2 + 4x = 7 + \sqrt{11}$

$$\sqrt{(x+2)^2} = \sqrt{11}$$

$$x + 2 = \pm \sqrt{11}$$

$$x = -2 \pm \sqrt{11}$$

(c) $x^{2/3} + 3x^{1/3} - 10 = 0$

$$\left(x^{1/3} + 5\right)\left(x^{1/3} - 2\right) = 0$$

$$x^{1/3} + 5 = 0$$

$$x^{1/3} = -5$$

$$x = -125$$

Note: $x = 2$ is also a solution, but it is not shown in the image.
d. \[ \sqrt{x+7} + 5 = x - 5 \]
\[ \sqrt{x+7} = x - 10 \]
\[ x+7 = (x-10)^2 \]
\[ x = 9, 2 \]

Only 9 checks, 2 does not.

32. (5 points) Rationalize the denominator and simplify.

\[ \frac{11\sqrt{3} - 22}{\sqrt{3} - 9} = \frac{11\sqrt{3} - 22}{\sqrt{3} - 9} \cdot \frac{\sqrt{3} + 9}{\sqrt{3} + 9} = -11\sqrt{3} + 22 \]
33. (5 points) The following function is one-to-one. 

\[ f(x) = \sqrt{x - 1}. \]

Find its inverse, \( f^{-1} \).

\[ f^{-1}(x) = \begin{array}{c} \frac{2}{x} + 1 \\ \end{array} \]

\[ y = \sqrt{x - 1} \]
\[ x = \sqrt{y - 1} \]
\[ x^2 = (\sqrt{y - 1})^2 \]
\[ x^2 = y - 1 \]
\[ x^2 + 1 = y \]
\[ y = x^2 + 1 \]

34. (5 points each) Perform the indicated operation and simplify. Leave your answer in radical form.

a. \[ \sqrt{6a^2b} \cdot \sqrt{2ab} \]
\[ \sqrt{6a^2b \cdot 2ab} \]
\[ \sqrt{3 \cdot 2 \cdot 2a^8b^2} \]
\[ 2ab \sqrt{3} \]

b. \[ 4\sqrt{54} + 3\sqrt{24} \]
\[ 4 \cdot 3\sqrt{6} + 3 \cdot 2 \sqrt{6} \]
\[ 12\sqrt{6} + 6\sqrt{6} \]
\[ 18\sqrt{6} \]

Assume that all variables represent positive real numbers.