MATH 109  
Spring 2014  

PART I: TRUE or FALSE

Circle TRUE or FALSE, whichever is correct. (2 pts. each)

1. \( x^2 - 9 = (x - 3)^2 = (x - 3)(x + 3) \)
   \( \text{TRUE} \) \( \text{FALSE} \)

2. If \(-3x > 6\) then \( x > -2 \)
   \( \frac{-3}{-3} \)
   \( x < -2 \)
   \( \text{TRUE} \) \( \text{FALSE} \)

3. The graph of \( y = x^2 \) is a parabola.
   \( \text{TRUE} \) \( \text{FALSE} \)

4. \( 36^{1/2} = 18 \)
   \( = \sqrt{36} = 6 \)
   \( \text{TRUE} \) \( \text{FALSE} \)

5. \( 2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3} \)
   \( \text{TRUE} \) \( \text{FALSE} \)

6. \( i^4 = -1 \)
   \( = 1 \)
   \( \text{TRUE} \) \( \text{FALSE} \)

PART II: FILL-IN-THE-BLANK (2 pts. each)

7. Express the following as a ratio in lowest terms.
   \( \frac{12}{5} : \frac{5}{5} \)
   \( \text{answer} \frac{2}{3} \)

8. The slope of any horizontal line is \( \boxed{\text{Zero}} \)
   \( \boxed{\text{Out}} \)
9. \( \sqrt{x^6} = \boxed{\begin{array}{c} x \\ 3 \end{array}} \)  \( (\sqrt{x})^2 = x \)

10. \( |6| = \boxed{6} \)

11. Change the following expression into a fractional exponent:
\[ \sqrt[3]{x^2} = \boxed{x^{\frac{2}{3}}} \]

12. Suppose that the line \( L \) is defined by the following equation.
\[ L : y = -\frac{2}{3}x + 5 \]
What is the slope of any line that is perpendicular to the line \( L \)?

13. To solve the equation \( x^2 - 6x = 5 \) by completing the square, the first step would be to add \( \boxed{\frac{9}{1}} \) to both sides.
\[ \left( \frac{1}{2}(-6) \right)^2 = \boxed{(\frac{9}{2})^2} = \boxed{q} \]

**PART III: MULTIPLE CHOICE**
Circle the letter of the correct answer. (2 pts. each)

14. Express \( x < 3 \) in interval notation.

(a) \((-\infty, 3]\)  (d) \((3, \infty)\)
(b) \((-\infty, 3)\)  (e) none of these
(c) \([3, \infty)\)

\[ \boxed{\text{A}} \]
15. Simplify the following expression.

\[ i^{10} = \left( i^2 \right)^5 = \left( -1 \right)^5 = -1 \]

(a) \( i \)  
(b) \(-1\)  
(c) \(-i\)  
(d) 1  
(e) none of these

PART IV: For problems 16-44, you must show all your work clearly on the exam for full credit. You must circle your final answer!!

16. (5 pts.) Factor the following polynomial completely.

\[ 5xy - 10x + 3y - 6 \]

\[ = 5(x - 2) + 3(y - 2) \]

\[ = (y - 2)(5x + 3) \]

17. (5 pts.) Reduce the following rational expression.

\[ \frac{x^2 - 2x - 15}{2x^2 - 9x - 5} = \frac{(x - 5)(x + 3)}{(2x + 1)(x - 5)} \]

\[ = \frac{x + 3}{2x + 1} \]
18. Perform the indicated operation and simplify.

(a) (5 pts.) \[ \frac{x^2 - 4}{6x^2} \div \frac{x^2 + 2x - 8}{3x^2 + 12x} = \frac{x^2 - 4}{6x^2} \cdot \frac{3x(x + 4)}{x^2 + 2x - 8} \]

\[ \frac{(x - 2)(x + 2)}{2 - 6x^2} \cdot \frac{3x(x + 4)}{(x + 4)(x^2 - 2)} \]

\[ = \frac{x(x + 2)}{2x} = \frac{x + 2}{2} \]

(b) (5 pts.) \[ \frac{8x + 3}{x + 2} - \frac{3x - 7}{x + 2} = \frac{(8x + 3) - (3x - 7)}{x + 2} = \frac{8x + 3 - 3x + 7}{x + 2} \]

\[ = \frac{5(x + 2)}{x + 2} = 5 \]

19. (5 pts.) Solve the following equation.

\[ 2 \left( \frac{x - 3}{2} \right) = x + 2 \]

\[ x - 3 = \frac{3x + 4}{2} \]

\[ -2x + 6 = 3x + 4 \]

\[-5x = 2 \]

\[ x = -\frac{2}{5} \]

\[ -1(-x) = (7) \]

\[ x = -7 \]
20. (5 pts.) Express the following phrase as a ratio in lowest terms.

\[
\frac{8 \text{ months}}{2 \text{ years}} = \frac{8 \div 4}{2 \div 2} = \frac{2}{1} = \frac{1}{2}
\]

8 months to 2 years \(\frac{x \times 12}{2} = \frac{8 \text{ months}}{2 \text{ years}}\)

21. (5 pts.) Solve the following equation.

\[
12 \left( \frac{2x + 3}{12} \right) = \frac{3}{4} \times 12^3
\]

\[
2x + 3 = \frac{9}{3} = 3
\]

\[
2x = 6
\]

\[
\underline{x = 3}
\]

22. (5 pts.) Solve the following inequality. Give the result in interval notation and graph the solution set.

\[
-3x + 4 < 10
\]

\[
-3x < 6
\]

\[
x > -2
\]

\(\text{See #2!}\)

Interval notation:

\((-2, \infty)\)

\(x > -2\)
23. (5 pts.) Solve the following equation.

\[ |2x - 1| - 5 = 2 \]
\[ +5 +5 \]
\[ 2x - 1 = \pm 7 \]
\[ 2x = 1 \pm 7 \]
\[ x = \frac{1 \pm 7}{2} \]
\[ \frac{8}{2} = 4 \]
\[ \frac{-6}{2} = -3 \]

24. (5 pts.) Solve the following inequality. Give the result in interval notation and graph the solution set.

\[ |x + 3| \leq 2 \]

\[ -2 \leq x + 3 \leq 2 \]
\[ -3 \leq x \leq -1 \]

25. (5 pts.) Use synthetic division to perform the division.

\[ (5x^2 + x - 2) \div (x + 1) \]

\[ -1 \]
\[ 5 \quad 1 \quad -2 \]
\[ -5 \quad 4 \]
\[ 5 \quad -4 \quad 2 \]

Quotient = \[ \frac{5x - 4}{2} \]

 Remainder = 2
6. Consider the following pair of points.

   \[ (3, -2) \text{ and } (1, 4) \]

   (5 pts.) Find the slope of the line passing through this pair of points.
   \[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-2)}{1 - 3} = \frac{0}{-2} = 0 \]

   (5 pts.) Write an equation of the line passing through this pair of points. Express your final answer using slope-intercept form.
   \[ y - y_1 = m(x - x_1) \]
   \[ y - (-2) = 0(x - 1) \]
   \[ y + 2 = 0 \]
   \[ y = -2 \]
   \[ -2 = -3(3) + b \]
   \[ -2 = -9 + b \]
   \[ b = 7 \]

27. Consider the following equation.

   \[ f(x) = 2x - 5 \]

   Find the following.
   (a) (2 pts.) \[ f(-2) \]
   \[ f(-2) = 2(-2) - 5 = -4 - 5 = -9 \]
   (b) (5 pts.) \[ f(w + 1) = 2(w + 1) - 5 \]
   \[ 2w + 2 - 5 \]
   \[ 2w - 3 \]
29. Consider the following equation.

\[ y = \frac{-2}{3}x + 1 \]

(a) (2 pts.) What is the slope? \(-\frac{2}{3}\)

(b) (2 pts.) What is the y-intercept? \((0, 1)\)

(c) (4 pts.) Using the slope and y-intercept found above, graph \(y = \frac{-2}{3}x + 1\) in the rectangular coordinate system below.

\[ (0,1) \]

\[ (3,-1) \]

29. (5 pts.) Find the distance between the following pair of points.

\(d = \sqrt{(9-3)^2 + (4 + 1)^2} = \sqrt{(6)^2 + (5)^2} = \sqrt{36 + 25} = \sqrt{61}\)

\((9, 4)\) and \((3, -1)\)
30. (a) (2 pts.) Graph the function $f(x) = x^2$

(b) (3 pts.) Graph $f(x) = x^2 + 1$ using a translation of $f(x) = x^2$

translation used up 1 unit

31. (5 pts.) Simplify the following expression.

$\left(25x^6\right)^{\frac{1}{3}} = \sqrt[3]{25x^6} = 5x^2$

32. (5 pts.) Simplify the following radical.

$2\sqrt[3]{x^2}$
33. (5 pts.) Perform the multiplication and simplify. Assume that all variables are positive.

\[
\sqrt{10x} \sqrt{5x^3 y^2} = \sqrt{2 \cdot 5x^4 y^2} = 5x^2 y \sqrt{2}
\]

34. (5 pts.) Simplify and combine like radicals. Express your answer in radical form.

\[
\frac{3\sqrt{18} + \sqrt{98}}{\sqrt{2} \cdot \sqrt{92}} \cdot 3 \sqrt{2} + 7 \sqrt{2} \cdot 9 \sqrt{2} + 7 \sqrt{2} = 16 \sqrt{2}
\]

35. (5 pts.) Rationalize the denominator and simplify.

\[
\frac{3}{5 - \sqrt{2}} \cdot \frac{(5 + \sqrt{2})}{(5 + \sqrt{2})} = \frac{15 + 3\sqrt{2}}{25 - 2}
\]
36. (5 pts.) Solve the following equation.

\[
\sqrt{11x + 3} - 3 = x + 3
\]

\[
\sqrt{11x + 3} = (x + 3)
\]

\[
\frac{11x + 3}{x + 3} = x^2 + 6x + 9
\]

\[
-x - 3
\]

\[
0 = x^2 - 5x + 6 = (x - 3)(x - 2)
\]

\[
3, 2
\]

They both check.

37. Perform the indicated operation. Write your final answer in the standard form: \(a + bi\).

(a) (5 pts.) \((2 - 3i) - (5 + 2i)\)

\[
2 - 3i - 5 - 2i
\]

\[
-3 - 5i
\]

(b) (5 pts.) \((2 - 3i)(5 + 2i)\)

\[
10 + 4i - 15i - 6i^2
\]

\[
10 - 11i - 6(-1)
\]

\[
10 - 11i + 6
\]

\[
16 - 11i
\]
38. (5 pts.) Solve the following equation by factoring.

\[ 2x^2 = 10x \]
\[ -10x - 10x \]
\[ 2x^2 - 10x = 0 \]
\[ 2x(x-5) = 0 \]
\[ \frac{2x}{2} = \frac{0}{2} \quad x = 0, 5 \]

39. (5 pts.) Solve the following equation using the square root method.

\[ (x - 3)^2 - 5 = 0 \]
\[ \sqrt{(x - 3)^2} = \pm \sqrt{5} \]
\[ x - 3 = \pm \sqrt{5} \]
\[ x = 3 \pm \sqrt{5} \]

40. (5 pts.) Solve the following equation by completing the square.

\[ x^2 - 8x = 1 \]
\[ +16 +16 \]
\[ x^2 - 8x + 16 = 17 \]
\[ \sqrt{(x-4)^2} = \sqrt{17} \]
\[ x - 4 = \pm \sqrt{17} \]
\[ x = 4 \pm \sqrt{17} \]
41. (5 pts.) Solve the following equation by the quadratic formula.

\[ x^2 - 4x + 2 = 0 \]

\[ x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)} = \frac{4 \pm \sqrt{16-8}}{2} = \frac{4 \pm \sqrt{8}}{2} \]

\[ = \frac{4}{2} \pm \frac{2\sqrt{2}}{2} = 2 \pm \sqrt{2} \]

42. (5 pts.) Solve the following equation. (Hint: try a substitution)

\[ x^{\frac{1}{4}} - 3x^{\frac{1}{4}} + 2 = 0 \]

\[ (x^{\frac{1}{4}} - 2)(x^{\frac{1}{4}} - 1) = 0 \]

\[ x^{\frac{1}{4}} - 2 = 0 \quad \text{or} \quad x^{\frac{1}{4}} - 1 = 0 \]

\[ x^{\frac{1}{4}} = 2 \quad \text{or} \quad x^{\frac{1}{4}} = 1 \]

\[ x = 16, \quad x = 1 \]

\[ \{16, 1\} \]
43. (5 pts.) **Solve** the following inequality. **Graph** the solution set on a real number line.

\[ x^2 + 2x - 3 > 0 \]
\[ (x + 3)(x - 1) = 0 \]
\[ x + 3 = 0 \quad x - 1 = 0 \]
\[ x = -3 \quad x = 1 \]

Test values:

\[ (1) \quad -3 \quad 0 \quad 1 \]

Inequality:
\[ x < -3 \quad \text{or} \quad x > 1 \]

Line graph:

\[ (-\infty, -3) \cup (1, \infty) \]

44. Consider the following pair of functions.

\[ f(x) = 3x + 2 \quad \text{and} \quad g(x) = x^2 - 5 \]

Find the following. Express your final answer without using parentheses.

(a) (5 pts.) \( f(g(x)) \)
\[ f\left(\frac{2}{x^2 - 5}\right) \]
\[ = 3\left(\frac{2}{x^2 - 5}\right) + 2 \]
\[ = 3\frac{2}{x^2 - 15} + 2 \]
\[ = 3x - 13 \]

(b) (5 pts.) \( f(g(2)) \)
\[ f\left(\frac{2}{2^2 - 5}\right) \]
\[ = f\left(\frac{2}{4 - 5}\right) \]
\[ = f(-1) \]
\[ = 3(-1) + 2 \]
\[ = -3 + 2 \]
\[ = -1 \]