Math 109 P-2 Instructor: Mihai Racovitan

Exam 1a on sections 5.1 – 6.7 **Solutions** February 8

1. Factor $x^3 + x^4$.

   Solutions :

   The common factor is x^3 : $x^3 + x^4 = x^3(1 + x)$. ■

2. Factor $2x^2 + 2xy - 3x - 3y$.

   Solutions :

   Group them like this : $2x^2 + 2xy - 3x - 3y = (2x^2 + 2xy) - (3x + 3y)$. Then

$$2x^2 + 2xy - 3x - 3y = \underbrace{(2x^2 + 2xy)}_{\text{factor } 2x} - \underbrace{(3x + 3y)}_{\text{factor } 3} = 2x(x + y) - 3(x + y) =$$

$$= \underbrace{2x(x + y) - 3(x + y)}_{\text{factor } (x+y)} = (x + y)(2x - 3) \quad . \quad ■$$

3. Factor $x^2 + 25$.

   Solutions :

   Do not make the following confusion. $x^2 - 25 = (x - 5)(x + 5)$, but here we have $x^2 + 25$, and this is PRIME ! ■

4. Factor $49x^2 - 16$.

   Solutions :

   Apply the formula $\boxed{F^2 - L^2 = (F - L)(F + L)}$.
   Here $49x^2$ and 16 are both squares: $49x^2 = (7x)^2$ and $16 = 4^2$. Therefore

$$49x^2 - 16 = (7x)^2 - 4^2 = (7x - 4)(7x + 4) \quad . \quad ■$$

5. Factor $x^4 - 81$.

   Solutions :

   Apply the formula $\boxed{F^2 - L^2 = (F - L)(F + L)}$ twice here .

Here x^4 and 81 are both squares: $x^4 = (x^2)^2$ and $81 = 9^2$. Therefore

$$x^4 - 81 = (x^2)^2 - 9^2 = (x^2 - 9)(x^2 + 9) \quad .$$

The second term, $x^2 + 9$, is prime because is the sum of two squares. But the first term, $x^2 - 9$, can be factored more:

$$x^4 - 81 = (x^2 - 9)(x^2 + 9) = (x^2 - 3^2)(x^2 + 9) = (x - 3)(x + 3)(x^2 + 9) \quad . \quad \blacksquare$$

6. Factor $4x^3 - 8x^2 - 32x$.

<u>*Solutions*</u> :

Pay attention first to the terms that there are in common, then try to factorize. Each term of $4x^3 - 8x^2 - 32x$ contains the factor $4x$. Factor it first:

$$4x^3 - 8x^2 - 32x = 4x(x^2 - 2x - 8) = 4x(x \pm \square)(x \pm \square) \quad .$$

The last term $x^2 - 2x - 8$ can be factored more (as I suggested above). Since the coefficient of x^2 is 1, then -8 (the free term) is the product and -2 (the coefficient of x) the sum.

$$-8 \left| \begin{array}{l} (1)(-8) \\ (-1)(8) \\ (2)(-4) \quad \rightarrow \quad 2 + (-4) = -2 \\ (-2)(4) \end{array} \right.$$

so,

$$4x^3 - 8x^2 - 32x = 4x(x^2 - 2x - 8) = 4x(x + 2)(x - 4) \quad . \quad \blacksquare$$

7. Factor $x^2 + 12x + 36$.

<u>*Solutions*</u> :

This is a perfect square polynomial. A perfect square like this is of the form:

$$(a + b)^2 = a^2 + 2ab + b^2$$

Our polynomial can be written as

$$x^2 + 12x + 36 = x^2 + 2 \cdot 6 \cdot x + 6^2 = (x + 6)^2 \quad . \quad \blacksquare$$

8. Factor $3x^2 - 5x - 2$.

<u>*Solutions*</u> :

Since the coefficient of x^2 is 3, then the product is -2 (the free term), but -5 is not the sum. You need trials. First, let me write the possible factorization

$$3x^2 - 5x - 2 = (3x \pm \square)(x \pm \square) \quad ,$$

and to decompose the product:

$$-2 \left| \begin{array}{l} (1)(-2) \\ (-1)(2) \end{array} \right.$$

Then, we have four trials:

$$(3x+1)(x-2) \quad , \quad (3x-2)(x+1) \quad , \quad (3x-1)(x+2) \quad , \quad (3x+2)(x-1)$$

The only one that works is

$$3x^2 - 5x - 2 = (3x+1)(x-2) \quad . \quad \blacksquare$$

9. Factor $4x^2 + 3x - 10$.

<u>*Solutions*</u> :

Since the coefficient of x^2 is 4, then the product is -10 (the free term), but $+3$ is not the sum. You need trials. First, let me write the possible factorizations

$$4x^2 + 3x - 10 = (4x \pm \square)(x \pm \square) \quad \text{, or} \quad 4x^2 + 3x - 10 = (2x \pm \square)(2x \pm \square) \quad ,$$

and to decompose the product:

$$-10 \left| \begin{array}{l} (1)(-10) \\ (-1)(10) \\ (2)(-5) \\ (-2)(5) \end{array} \right.$$

Then, we have twelve trials, eight for $(4x \pm \square)(x \pm \square)$ and four for $(2x \pm \square)(2x \pm \square)$. Let me write the only one that works:

$$4x^2 + 3x - 10 = (4x-5)(x+2) \quad . \quad \blacksquare$$

10. Solve $3x^2 + 8x = 0$.

<u>*Solutions*</u> :

This is an equation. You need to find the value of x that satisfies the above equation. To solve, we factor first, and then we apply the zero-factor property:

$$3x^2 + 8x = 0 \Rightarrow x(3x+8) = 0 \Rightarrow x = 0 \text{ or } 3x + 8 = 0 \Rightarrow x = 0 \text{ or } 3x = -8 \Rightarrow$$

$$\Rightarrow x = 0 \text{ or } x = \frac{-8}{3} \Rightarrow x = 0 \text{ or } x = -\frac{8}{3}$$

The solutions of the equations are $\boxed{x = 0}$ and $\boxed{x = -\frac{8}{3}}$. ■

11. Express the phrase as a ratio in lowest terms: 45 minutes to 1 hour.

<u>*Solutions*</u> :

Convert the hour in minutes: 1 hour = 60 minutes. Then, the ratio is

$$\frac{45}{60} = \frac{3 \cdot 15}{4 \cdot 15} = \frac{3 \cdot \cancel{15}}{4 \cdot \cancel{15}} = \frac{3}{4} \text{ (three quarters)} \quad . \quad \blacksquare$$

12. A school board determines that there should be 3 teachers for every 70 students. How many teachers are needed for an enrollment of 2310 students?

<u>*Solutions*</u> :

You should ask first the question: what is the ratio of teachers per student: $\frac{3}{70}$ teachers per each student. (Using a calculator to find out what is 3/70 is not recommended here, and you will see why)

Then, for 2310 students, we have

$$2310 \cdot \left(\frac{3}{70}\right) = \frac{2310 \cdot 3}{70} = \frac{6930}{70} = 99 \text{ teachers} \ . \ \blacksquare$$

13. Solve $\dfrac{2x-1}{16} = \dfrac{20}{64}$.

<u>*Solutions*</u> :

Since $64 = 4 \cdot 16$, multiply both sides by 64:

$$64 \cdot \frac{2x-1}{16} = 64 \cdot \frac{20}{64} \Rightarrow \frac{64 \cdot (2x-1)}{16} = \frac{64 \cdot 20}{64} \Rightarrow \frac{\cancel{64}^{4} \cdot (2x-1)}{\cancel{16}} = \frac{\cancel{64} \cdot 20}{\cancel{64}} \Rightarrow$$

$$\Rightarrow 4(2x-1) = 20 \Rightarrow 8x - 4 = 20 \Rightarrow 8x = 20 + 4 \Rightarrow 8x = 24 \Rightarrow \frac{8x}{8} = \frac{24}{8} \Rightarrow x = 3 \quad .$$

The solution of the equation is $\boxed{x = 3}$. ■

14. Simplify the rational expression $\dfrac{x^2 - 5x - 36}{3x^2 + 11x - 4}$.

<u>*Solutions*</u> :

We have to factor both numerator and denominator, and then simplify.

First, factor $x^2 - 5x - 36$. Since the coefficient of x^2 is 1, then the product is -36, and the sum is -5. Here are the possibilities:

$$\begin{array}{r|l}
-36 & (1)(-36) \\
 & (-1)(36) \\
 & (2)(-18) \\
 & (-2)(18) \\
 & (3)(-12) \\
 & (-3)(12) \\
 & (4)(-9) \quad \rightarrow \quad 4 + (-9) = -5 \\
 & (-4)(9) \\
 & (6)(-6)
\end{array}$$

so the solution that works is $x^2-5x-36 = (x+4)(x-9)$. We suppose to factor $3x^2+11x-4$ too, but let us use a shortcut. May be one of the factors $x + 4$ and $x - 9$ is also a factor of $3x^2 + 11x - 4$:

$$(x + 4)(3x - 1) = 3x^2 - x + 12x - 4 = 3x^2 + 11x - 4 \text{ actually works !}$$

Then

$$\frac{x^2 - 5x - 36}{3x^2 + 11x - 4} = \frac{(x + 4)(x - 9)}{(x + 4)(3x - 1)} = \frac{\cancel{(x+4)}\ (x - 9)}{\cancel{(x+4)}\ (3x - 1)} = \frac{x - 9}{3x - 1} \quad . \quad \blacksquare$$

15. Do the operation and simplify $\dfrac{7x - 28}{7x + 42} \cdot \dfrac{x + 6}{x^2 - 16}$.

<u>*Solutions*</u> :

Do not simplify over the + and − signs! Factor, and simplify over the product:

$$\frac{7x - 28}{7x + 42} \cdot \frac{x + 6}{x^2 - 16} = \frac{7(x - 4)}{7(x + 6)} \cdot \frac{x + 6}{(x - 4)(x + 4)} = \frac{7(x - 4)(x + 6)}{7(x + 6)(x - 4)(x + 4)} =$$

$$= \frac{\cancel{7}\cancel{(x-4)}\ \cancel{(x+6)}}{\cancel{7}\cancel{(x+6)}\ \cancel{(x-4)}\ (x + 4)} = \frac{1}{x + 4} \quad . \quad \blacksquare$$

16. Do the operation and simplify $\dfrac{18w^2}{5y^3} \div \dfrac{9w^5}{30y}$.

<u>*Solutions*</u> :

Is a division, and we have the rule $\boxed{\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \cdot \dfrac{d}{c} = \dfrac{a \cdot d}{b \cdot c}}$.

Then

$$\frac{18w^2}{5y^3} \div \frac{9w^5}{30y} = \frac{18w^2}{5y^3} \cdot \frac{30y}{9w^5} = \frac{\cancel{18}^{2} \ \cancel{w^2}}{\cancel{5} \ \cancel{y^3}^{\,2}} \cdot \frac{\cancel{30}^{6} \ \cancel{y}}{\cancel{9} \ \cancel{w^5}^{\,3}} = \frac{2 \cdot 6}{y^2 \cdot w^3} = \frac{12}{y^2 w^3} \quad . \quad \blacksquare$$

17. Do the operation and simplify $\dfrac{x^2-36}{x-6} \div \dfrac{2x}{4x+24}$.

<u>*Solutions*</u> :

Is a division, and we have the rule $\boxed{\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \cdot \dfrac{d}{c} = \dfrac{a \cdot d}{b \cdot c}}$.

Then

$$\frac{x^2-36}{x-6} \div \frac{2x}{4x+24} = \frac{x^2-36}{x-6} \cdot \frac{4x+24}{2x} = \frac{(x+6)(x-6)}{x-6} \cdot \frac{4(x+6)}{2x} =$$

$$= \frac{(x+6)\cancel{(x-6)}}{\cancel{x-6}} \cdot \frac{\cancel{4}(x+6)}{\cancel{2}x} = \frac{(x+6) \cdot 2(x+6)}{x} = \frac{2(x+6)^2}{x} \quad . \quad \blacksquare$$

18. Do the operation and simplify $\dfrac{6x+7}{x+4} - \dfrac{3x-5}{x+4}$.

<u>*Solutions*</u> :

These fractions have the same denominator: $\dfrac{a}{b} - \dfrac{c}{b} = \dfrac{a-c}{b}$. Then

$$\frac{6x+7}{x+4} - \frac{3x-5}{x+4} = \frac{(6x+7)-(3x-5)}{x+4} = \frac{6x+7-3x+5}{x+4} = \frac{3x+12}{x+4} =$$

$$= \frac{3(x+4)}{x+4} = \frac{3\cancel{(x+4)}}{\cancel{x+4}} = 3 \quad . \quad \blacksquare$$

19. Do the operation and simplify $\dfrac{4}{x+2} - \dfrac{8}{x^2-4} - \dfrac{x}{2-x}$.

<u>*Solutions*</u> :

These three fractions have different denominator, so you need to find the common denominator, L.C.M. Here, x^2-4 is the L.C.M. of all three denominators. But you have to make an observation: the last denominator is $2-x = -(x-2)$. Let us rewrite the expression

$$\frac{4}{x+2} - \frac{8}{x^2-4} - \frac{x}{2-x} = \frac{4}{x+2} - \frac{8}{x^2-4} - \frac{x}{-(x-2)} = \frac{4}{x+2} - \frac{8}{x^2-4} + \frac{x}{x-2}$$

The common denominator of these three denominators, $x+2$, $x^2-4=(x-2)(x+2)$, and $x-2$, is $(x-2)(x+2)$. Therefore,

$$\frac{4(x-2)}{(x+2)(x-2)}-\frac{8}{(x-2)(x+2)}+\frac{x(x+2)}{(x-2)(x+2)}=\frac{4(x-2)-8+x(x+2)}{(x-2)(x+2)}=$$

$$=\frac{4x-8-8+x^2+2x}{(x-2)(x+2)}=\frac{x^2+6x-16}{(x-2)(x+2)}=\frac{(x-2)(x+8)}{(x-2)(x+2)}=\frac{x+8}{x+2} \quad . \quad \blacksquare$$

20. Solve $\dfrac{2x+3}{2}+\dfrac{x+4}{3}=\dfrac{10x+13}{6}$.

Solutions :

The common denominator is 6. Multiply both sides by 6:

$$6\cdot\left(\frac{2x+3}{2}+\frac{x+4}{3}\right)=\frac{10x+13}{6}\cdot 6 \Rightarrow 6\cdot\frac{2x+3}{2}+6\cdot\frac{x+4}{3}=10x+13 \Rightarrow$$

$\Rightarrow 3(2x+3)+2(x+4)=10x+13 \Rightarrow 6x+9+2x+8=10x+13 \Rightarrow 8x+17=10x+13 \Rightarrow$

$\Rightarrow 10x+13=8x+17 \Rightarrow 10x=8x+17-13 \Rightarrow 10x=8x+4 \Rightarrow 10x-8x=4 \Rightarrow 2x=4 \Rightarrow x=2$

The solution of our equation is $\boxed{x=2}$. $\blacksquare$