Math 109 P-2 Review 05/21/2008

Instructor: Mihai Racovitan **Math 109 Formulas**

1. Formulas for Exponential and Power Expressions

$$1^a = 1 \text{ for any } a \quad \text{, and} \quad a^0 = 1 \text{ if } a \neq 0$$

$$x^{a+b} = x^a \cdot x^b$$

$$x^{a-b} = \frac{x^a}{x^b}$$

$$x^{a \cdot b} = (x^a)^b$$

$$x^a \cdot y^a = (x \cdot y)^a$$

$$\frac{x^a}{y^a} = \left(\frac{x}{y}\right)^a$$

2. Formulas for Radical Expressions

$$\sqrt[n]{1} = 1 \text{ for any } n \geq 2 \quad \text{, and} \quad \sqrt[n]{0} = 0 \text{ for any } n \geq 2$$

$$\sqrt[n]{-1} \text{ undefined if } n \text{ even} \quad \text{, and} \sqrt[n]{-1} = -1 \text{ if } n \text{ odd}$$

$$\sqrt[n]{x} \cdot \sqrt[n]{y} = \sqrt[n]{x \cdot y}$$

$$\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}} \text{ , if } y \neq 0$$

3. Radical Expressions Converted to Exponential Expressions

$$\sqrt[n]{x} = x^{1/n}$$

$$\sqrt[n]{x^n} = \begin{cases} x, & \text{if } n \text{ is odd;} \\ |x|, & \text{if } n \text{ is even.} \end{cases}$$

$$\left(\sqrt[n]{x}\right)^n = x$$

$$\sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m = x^{m/n}$$

4. Negative Powers for Exponential and Radical Expressions

$$x^{-n} = \frac{1}{x^n}$$

$$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$$

$$x^{-1/n} = \frac{1}{\sqrt[n]{x}}$$

$$x^{-m/n} = \frac{1}{\sqrt[n]{x^m}} = \frac{1}{(\sqrt[n]{x})^m}$$

$$\left(\frac{x}{y}\right)^{-1/n} = \sqrt[n]{\frac{y}{x}}$$

5. Formulas for Factoring Special Polynomials

5.1. **Difference of two squares.**

$$x^2 - a^2 = (x - a) \cdot (x + a)$$

5.2. **Sum of two squares.**

$$x^2 + a^2 \quad \text{is prime, cannot be factored}$$

5.3. **Difference of two cubes.**

$$x^3 - a^3 = (x - a) \cdot (x^2 + a \cdot x + a^2)$$

5.4. **Sum of two cubes.**

$$x^3 + a^3 = (x + a) \cdot (x^2 - a \cdot x + a^2)$$

5.5. **Zero-factor Property.**

$$\text{If } a \cdot b = 0 \text{ , then } a = 0 \text{ or } b = 0$$

6. Formulas for Expanding Special Polynomials

6.1. **The Square of the Sum.**

$$(x + a)^2 = x^2 + 2ax + a^2$$

6.2. **The Square of the Difference.**

$$(x - a)^2 = x^2 - 2ax + a^2$$

6.3. **The Cube of the Sum.**

$$(x+a)^3 = x^3 + 3ax^2 + 3a^2x^2 + a^3$$

6.4. **The Cube of the Difference.**

$$(x-a)^3 = x^3 - 3ax^2 + 3a^2x - a^3$$

7. Equations and Inequalities with Absolute Values

If $x \geq 0$, then $\mid x \mid = x$.

If $x < 0$, then $\mid x \mid = -x$.

If $k > 0$, then

$$\begin{array}{l} \mid x \mid = k \text{ is equivalent to } x = k \text{ or } x = -k \\ \mid x \mid < k \text{ is equivalent to } -k < x < k \\ \mid x \mid > k \text{ is equivalent to } x < -k \text{ or } x > k \end{array}$$

8. Complex Numbers

8.1. **The Definition of i .**

$$\sqrt{-1} = i \quad , \quad i^2 = -1$$

If a is strictly positive, then

$$\sqrt{-a} = \mathbf{i} \cdot \sqrt{a} = \sqrt{a}\ \mathbf{i}$$

8.2. **Sum of Two Complex Numbers.**

$$(a + b\mathbf{i}) + (c + d\mathbf{i}) = (a + c) + (b + d)\mathbf{i}$$

8.3. **Difference of Two Complex Numbers.**

$$(a + b\mathbf{i}) - (c + d\mathbf{i}) = (a - c) + (b - d)\mathbf{i}$$

8.4. **Multiplication of Two Complex Numbers.**

$$(a + b\mathbf{i}) \cdot (c + d\mathbf{i}) = (ac - bd) + (ad + bc)\mathbf{i}$$

8.5. **The Length of a Complex Number or its Absolute Value.**

$$\mid a + b\mathbf{i} \mid = \sqrt{a^2 + b^2}$$

8.6. **The Conjugate of a Complex Number.**

The conjugate of $a + b\mathbf{i}$ is $a - b\mathbf{i}$

8.7. **Multiplying Two Conjugate Complex Numbers.**

$$(a + b\mathbf{i}) \cdot (a - b\mathbf{i}) = a^2 + b^2$$

9. Distance Formula between Two Points

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by the formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

10. Lines and slopes

The slope of the line passing through the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of a horizontal line $y = a$ is $m = 0$, while the slope of a vertical line $x = b$ is undefined.

If the slope of the line $y = mx + b$ is strictly positive, the line is strictly increasing. If the slope of the line $y = mx + b$ is strictly negative, the line is strictly decreasing.

The slope-intercept form of a line is

$$y = mx + b \quad ,$$

where m is the slope, and b is the y intercept.

The point-slope form of a line is

$$y - y_1 = x - x_1 \quad ,$$

where m is the line's slope, and (x_1, y_1) is a given point on the line.

Two lines with the same slope are parallel. If the product of the slopes of two lines is -1, the lines are perpendicular.

11. Solving Quadratic Equations

The quadratic equations are of the form $ax^2 + bx + c = 0$. If you can factor $ax^2 + bx + c$ as a product of two terms, then you can use the 0-property to solve it. Let see other methods to solve it.

11.1. Solving equations of the form $\mathbf{x^2 = a}$.

$$x^2 = a \Rightarrow \sqrt{x^2} = \sqrt{a} \Rightarrow \mid x \mid = \sqrt{a} \Rightarrow x = \sqrt{a} \text{ or } x = -\sqrt{a}$$

11.2. **Completing the square.** Consider the equation $x^2 + bx + c = 0$ (the coefficient of x^2 has to be 1). Then, you complete the square:

$$x^2+bx+c=0 \Rightarrow x^2+bx=-c \Rightarrow x^2+bx+\left(\frac{b}{2}\right)^2=\left(\frac{b}{2}\right)^2-c \Rightarrow \left(x+\frac{b}{2}\right)^2=\left(\frac{b}{2}\right)^2-c \Rightarrow$$

$$\Rightarrow \sqrt{\left(x+\frac{b}{2}\right)^2}=\sqrt{\left(\frac{b}{2}\right)^2-c} \Rightarrow \left|x+\frac{b}{2}\right|=\sqrt{\left(\frac{b}{2}\right)^2-c} \Rightarrow x+\frac{b}{2}=\pm\sqrt{\left(\frac{b}{2}\right)^2-c} \Rightarrow$$

$$\Rightarrow x=-\frac{b}{2}+\sqrt{\left(\frac{b}{2}\right)^2-c} \quad \text{or} \quad x=-\frac{b}{2}-\sqrt{\left(\frac{b}{2}\right)^2-c}$$

11.3. **Quadratic formula.** The solutions of the quadratic equation $ax^2 + bx + c = 0$ are :

$$x_1=\frac{-b+\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a} \quad , \quad x_2=\frac{-b-\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}$$

The discriminant $\Delta = b^2 - 4ac$:

If	then the roots of $ax^2 + bx + c$ are
$b^2 - 4ac > 0$	real and unequal
$b^2 - 4ac = 0$	real and equal
$b^2 - 4ac < 0$	complex and conjugate