Math 109 P Section 2 — Quiz 11 A

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Solutions

(1) Solve the equation by using the quadratic formula. $x^2 + 3x + 3 = 0$

Solutions.

The quadratic formula states that the solutions of the quadratic equation

$$ax^2 + bx^2 + c = 0$$

are given by :

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad . \tag{1}$$

In our case, the coefficients of $x^2 + 3x + 3 = 0$ are: $a = 1$, $b = 3$, and $c = 3$. Then, by (1), we have that

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-3 + \sqrt{3^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{-3 + \sqrt{9 - 12}}{2} = \frac{-3 + \sqrt{-3}}{2} = \frac{-3 + \sqrt{3}\ \mathbf{i}}{2} \quad .$$

The other solution is :

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-3 - \sqrt{3^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{-3 - \sqrt{3}\ \mathbf{i}}{2} \quad .$$

The solutions are complex and conjugate : $x_1 = \dfrac{-3 + \sqrt{3}\ \mathbf{i}}{2}$ and $x_2 = \dfrac{-3 - \sqrt{3}\ \mathbf{i}}{2}$. ■

(2) Use the discriminant $\Delta = b^2 - 4ac$ to determine what type of solutions exist for the quadratic equation $4x^2 + 4x + 1 = 0$. Do not solve the equation.

Solutions.

You do NOT need to solve the equation! You need just to check the value of Δ and its sign. The coefficients of the equation $4x^2 + 4x + 1 = 0$ are $a = 4$, $b = 4$ and $c = 1$. Therefore,

$$\Delta = b^2 - 4ac = 4^2 - 4 \cdot 4 \cdot 1 = 16 - 16 = 0 \quad .$$

Therefore, the solutions are real and equal because $\Delta = 0$. ■

(3) Solve the equation. $x^4 - 6x^2 + 5 = 0$

Solutions.

This equation cannot be solved directly. We need a substitution that transform this equation in a quadratic equation. The variable x appears with the 2nd and the 4th powers in

$$x^4 - 6x^2 + 5 = 0 \tag{2}$$

But, $(x^2)^2 = x^4$. This suggest the substitution $y = x^2$ in $x^4 - 6x^2 + 5 = 0$ (observe that $x^4 = (x^2)^2 = y^2$) :

$$y^2 - 6y + 5 = 0 \tag{3}$$

The last equation is a quadratic equation and we can use the quadratic formula (1) to solve it. The coefficients are $a = 1$, $b = -6$ and $c = 5$. Then

$$y_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) + \sqrt{(-6)^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1} = \frac{6 + \sqrt{36 - 20}}{2} =$$

$$= \frac{6 + \sqrt{16}}{2} = \frac{6 + 4}{2} = \frac{10}{2} = 5 \quad .$$

The second solution is:

$$y_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) - \sqrt{(-6)^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1} = \frac{6 - \sqrt{16}}{2} = \frac{6 - 4}{2} = \frac{2}{2} = 1 \quad .$$

We are not done! We just solved equation (3), but our objective is to solve equation (2). Then, we go back to the substitution $y = x^2$:

$$y_1 = 5 \text{ and } y = x^2 \Rightarrow x^2 = 5 \Rightarrow \sqrt{x^2} = \sqrt{5} \Rightarrow |x| = \sqrt{5} \Rightarrow x = \sqrt{5} \text{ or } x = -\sqrt{5} \quad ,$$

and

$$y_2 = 1 \text{ and } y = x^2 \Rightarrow x^2 = 1 \Rightarrow \sqrt{x^2} = \sqrt{1} \Rightarrow |x| = 1 \Rightarrow x = 1 \text{ or } x = -1 \quad .$$

Therefore, we have four solutions : $x_1 = \sqrt{5}$, $x_2 = -\sqrt{5}$, $x_3 = 1$ and $x_4 = -1$. ■

(4) Solve the equation. $2x - \sqrt{x} - 3 = 0$

Solutions.

This equation cannot be solved directly. We need a substitution that transform this equation in a quadratic equation. The variable x appears within $\sqrt{x}$ and with the 1st power in

$$2x - \sqrt{x} - 3 = 0 \tag{4}$$

But, $(\sqrt{x})^2 = x$. This suggest the substitution $y = \sqrt{x}$ in $2x - \sqrt{x} - 3 = 0$ (observe that $x = (\sqrt{x})^2 = y^2$) :

$$2y^2 - y - 3 = 0 \tag{5}$$

The last equation is a quadratic equation and we can use the quadratic formula (1) to solve it. The coefficients are $a = 2$, $b = -1$ and $c = -3$. Then

$$y_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) + \sqrt{(-1)^2 - 4 \cdot 2 \cdot (-3)}}{2 \cdot 2} = \frac{1 + \sqrt{1 + 24}}{4} =$$

$$= \frac{1 + \sqrt{25}}{4} = \frac{1 + 5}{4} = \frac{6}{4} = \frac{3}{2} \quad ,$$

and

$$y_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) - \sqrt{(-1)^2 - 4 \cdot 2 \cdot (-3)}}{2 \cdot 2} = \frac{1 - \sqrt{25}}{4} = \frac{1 - 5}{4} = \frac{-4}{4} = -1 \quad .$$

We are not done! We just solved equation (5), but our objective is to solve equation (4). Then, we go back to the substitution $y = \sqrt{x}$:

$$y_1 = \frac{3}{2} \text{ and } y = \sqrt{x} \Rightarrow \sqrt{x} = \frac{3}{2} \Rightarrow (\sqrt{x})^2 = \left(\frac{3}{2}\right)^2 \Rightarrow x = \frac{9}{4} \quad .$$

The other solution $y_2 = -1$ cannot be accepted as a solution because it implies that $y = \sqrt{x} = -1$, which is not accepted (for example, $\sqrt{4} = 2$, not -2) .

The only acceptable solution is $x = \dfrac{9}{4}$. ■