Math 109 P Section 2 — Quiz 1 A

TA	:	Racovitan Michael
Office	:	DU 370
E-mail	:	racovita@math.niu.edu
Web page	:	www.math.niu.edu/∼racovita/Math109P/Math109P2.html

Solutions

(1) Simplify. $(2x - y)(x + 3y)$

Solutions.

$(2x-y)(x+3y) = (2x)(x)+(2x)(3y)-(y)(x)-(y)(3y) = 2x^2+6xy-xy-3y^2 = 2x^2+5xy-3y^2$. ■

(2) Factor into prime components. $3x^2y^3 - 9x^4y^3z$

Solutions.

Both terms, $3x^2y^3$ and $9x^4y^3z$ have the following common factors : 3, x^2, and y^3. Then

$$3x^2y^3 - 9x^4y^3z = 3x^2y^3\left(1 - 3x^2z\right) \quad . \quad ■$$

(3) Factor into prime components. $9a^2b^2 - 16b^4$

Solutions.

Both terms, $9a^2b^2$ and $16b^4$ have the following common factor : b^2. Factor it first.

$$9a^2b^2 - 16b^4 = b^2\left(9a^2 - 16b^2\right) \quad .$$

Now, both $9a^2$ and $16b^2$ are perfect squares: $9a^2 = (3a)^2$ and $16b^2 = (4b)^2$. Now, use the formula

$$F^2 - L^2 = (F - L)(F + L)$$

Then

$$9a^2b^2 - 16b^4 = b^2\left(9a^2 - 16b^2\right) = b^2\left(3a - 4b\right)\left(3a - 4b\right) \quad . \quad ■$$

(4) Factor into prime components. $24a + 4ax - 4ax^2$

Solutions.

Write $24a+4ax-4ax^2$ in descending powers of x : $24a+4ax-4ax^2 = -4ax^2+4ax+24a$.

As you can see, all three terms $-4ax^2$, $4ax$, and $24a$ have in common $4a$. We factor also the $-$, so we have to change the sign.

$$-4ax^2 + 4ax + 24a = -4a\left(x^2 - x - 6\right) = -4a(x \pm \square)(x \pm \square)$$

Now, the free term of $x^2 - x - 6$ is -6, and this is the product of the following pairs: $(1, -6)$, $(-1, 6)$, $(2, -3)$, $(-2, 3)$.

The only pair that sums up to -1 (the coefficient of x in x^2-x-6) is $(2, -3)$. Therefore,

$$-4ax^2 + 4ax + 24a = -4a\left(x^2 - x - 6\right) = -4a(x + 2)(x - 3) \quad . \quad ■$$