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 Web page : [www.math.niu.edu/~racovita/Math109P/Math109P2.html](http://www.math.niu.edu/~racovita/Math109P/Math109P2.html)

## Solutions

(1) Simplify :  $\frac{x(x+3) - 3(x-1)}{x^2+3}$  .

Solutions.

Simplify the numerator first, pay attention to the rule of signs, and then try to simplify the fraction:

$$\begin{aligned} \frac{x(x+3) - 3(x-1)}{x^2+3} &= \frac{x \cdot x + x \cdot 3 - 3 \cdot x + 3 \cdot 1}{x^2+3} = \frac{x^2 + 3x - 3x + 3}{x^2+3} = \\ &= \frac{x^2 + \cancel{3x} - \cancel{3x} + 3}{x^2+3} = \frac{x^2+3}{x^2+3} = \frac{(\cancel{x^2}/\cancel{3})}{(\cancel{x^2}/\cancel{3})} = 1 \quad \blacksquare \end{aligned}$$

(2) Divide.  $\frac{x^2-4}{3x+6} \div \frac{x-2}{x+2}$

Solutions.

The rule of dividing fractions states that :  $\boxed{\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}}$  .

Then, in our case, use the above rule, multiply the fractions, factor terms, and then simplify:

$$\begin{aligned} \frac{x^2-4}{3x+6} \div \frac{x-2}{x+2} &= \frac{x^2-4}{3x+6} \cdot \frac{x+2}{x-2} = \frac{(x^2-4) \cdot (x+2)}{(3x+6) \cdot (x-2)} = \\ &= \frac{(x-2)(x+2)(x+2)}{3(x+2)(x-2)} = \frac{(\cancel{x-2}) (\cancel{x+2}) (x+2)}{3 (\cancel{x-2}) (\cancel{x+2})} = \frac{x+2}{3} \quad \blacksquare \end{aligned}$$

(3) Simplify.  $\frac{x+3}{x^2} + \frac{x+5}{2x}$

Solutions.

The fractions  $\frac{x+3}{x^2}$  and  $\frac{x+5}{2x}$  don't have the same denominator, so we need to find first the common denominator, i.e. the least common multiple of the denominators. Write them as following

$$\begin{aligned} x^2 &= x \cdot x \\ 2x &= 2 \cdot x \\ \text{L.C.M.} &= \underbrace{x \cdot x}_{\text{twice}} \cdot 2 = 2x^2 \end{aligned}$$

Therefore, the common denominator is  $2x^2$ . We multiply each fractions with what is missing:

$$\frac{x+3}{x^2} + \frac{x+5}{2x} = \frac{(x+3) \cdot 2}{x^2 \cdot 2} + \frac{(x+5) \cdot x}{2x \cdot x} = \frac{2(x+3)}{2x^2} + \frac{x(x+5)}{2x^2} .$$

Now both fractions have the same denominator. Add them according to the rule:

$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$  . Then, simplify.

$$\begin{aligned} \frac{2(x+3)}{2x^2} + \frac{x(x+5)}{2x^2} &= \frac{2(x+3) + x(x+5)}{2x^2} = \frac{2x+6+x^2+5x}{2x^2} = \\ &= \frac{x^2+7x+6}{2x^2} = \frac{(x+1)(x+6)}{2x^2} . \quad \blacksquare \end{aligned}$$