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Solutions

(1) Solve the inequality. Graphic and interval solutions, please. $|4x - 1| \leq 7$

Solutions.

We have the following fact.

$$\boxed{|x| \leq k \text{ is equivalent to } -k \leq x \leq k \quad (k \geq 0)} \quad (1)$$

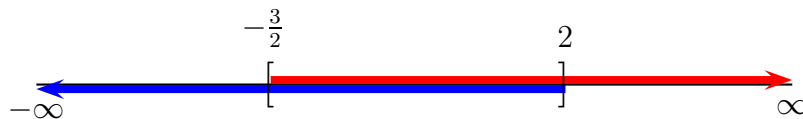
Therefore, we have, by (1), that

$$|4x - 1| \leq 7 \text{ is equivalent to } -7 \leq 4x - 1 \leq 7$$

We can solve this set of inequalities together:

$$\underbrace{-7}_{+1} \leq \underbrace{4x - 1}_{+1} \leq \underbrace{7}_{+1} \Rightarrow -6 \leq 4x \leq 8 \Rightarrow \frac{-6}{4} \leq \frac{4x}{4} \leq \frac{8}{4} \Rightarrow -\frac{3}{2} \leq x \leq 2 \quad .$$

Here is the graphic solution:



We now see what the red line and blue line have in common: $x \in \left[-\frac{3}{2}, 2\right]$. ■

(2) Solve the inequality. Graphic and interval solutions, please. $\left|\frac{x-4}{3}\right| > 2$

Solutions.

We have the following fact. If $k \geq 0$, then

$$\boxed{|x| > k \text{ is equivalent to } x < -k \text{ or } x > k} \quad (2)$$

Therefore, we have, by (2), that

$$\left| \frac{x-4}{3} \right| > 2 \text{ is equivalent to } \frac{x-4}{3} < -2 \text{ or } \frac{x-4}{3} > 2$$

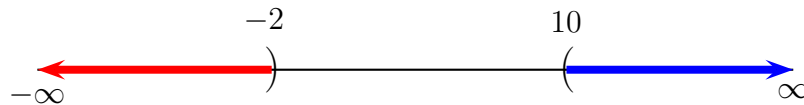
We cannot solve this set of inequalities together, we solve them separately. The first inequality is

$$\frac{x-4}{3} < -2 \Big/ \cdot 3 \Rightarrow x-4 < -6 \Rightarrow \underbrace{x-4}_{+4} < \underbrace{-6}_{+4} \Rightarrow x < -2 \quad .$$

The second inequality is

$$\frac{x-4}{3} > 2 \Big/ \cdot 3 \Rightarrow x-4 > 6 \Rightarrow \underbrace{x-4}_{+4} > \underbrace{6}_{+4} \Rightarrow x > 10 \quad .$$

The solution is given by $x < -2$ **OR** $x > 10$. Here is a graphic solution .



These interval solutions are considered together : $x \in (-\infty, -2) \cup (10, \infty)$. ■

(3) Factor. $3x^3 - 48x$

Solutions.

We have the following formula for factoring the difference of two squares:

$$\boxed{F^2 - L^2 = (F - L)(F + L)} \quad (3)$$

Factor first what they have in common:

$$3x^3 - 48x = 3x(x^2 - 16) = 3x(x^2 - 4^2) \stackrel{(3)}{=} 3x(x-4)(x+4) \quad . \quad \blacksquare$$

(4) Factor. $x^3 - 27$

Solutions.

We have the following formula for factoring the difference of two cubes :

$$\boxed{a^3 - b^3 = (a - b)(a^2 + ab + b^2)} \quad (4)$$

$$x^3 - 27 = x^3 - 3^3 \stackrel{(4)}{=} (x-3)(x^2 + 3x + 9) \quad . \quad \blacksquare$$