Math 109 P Section 2 — Quiz 8 A

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Solutions

(1) Simplify each expression.

(a) $4^{1/2}$ (b) $\left(\frac{1}{4}\right)^{1/2}$ (c) $(-125)^{1/3}$ (d) $\left(\frac{27}{8}\right)^{-4/3}$

Solutions.

(a) We have the formula

$$\boxed{a^{1/n} = \sqrt[n]{a}} \tag{1}$$

Then, according to (1): $4^{1/2} = \sqrt{4} = 2$. ■

(b) We have the formula

$$\boxed{\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}} \tag{2}$$

Then, according to (1) and (2) we have

$$\left(\frac{1}{4}\right)^{1/2} \overset{(1)}{=} \sqrt{\frac{1}{4}} \overset{(2)}{=} \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2} \quad . \quad ■$$

(c) We use the formula (1):

$$(-125)^{1/3} = \sqrt[3]{-125} = -5 \quad . \quad ■$$

(d) We have the formulas

$$\boxed{\left(\frac{a}{b}\right)^{-m/n} = \left(\frac{b}{a}\right)^{m/n} = \left(\sqrt[n]{\frac{b}{a}}\right)^{m}} \tag{3}$$

and

$$\boxed{\left(\frac{a}{b}\right)^{n} = \frac{a^n}{b^n}} \tag{4}$$

Therefore, we use (3), (2), and (4):

$$\left(\frac{27}{8}\right)^{-4/3} \overset{(3)}{=} \left(\frac{8}{27}\right)^{4/3} = \left(\sqrt[3]{\frac{8}{27}}\right)^{4} \overset{(2)}{=} \left(\frac{\sqrt[3]{8}}{\sqrt[3]{27}}\right)^{4} = \left(\frac{2}{3}\right)^{4} \overset{(4)}{=} \frac{2^4}{3^4} = \frac{16}{81} \quad . \quad ■$$

(2) Perform the operations. Write your answer without negative exponents.

$$\frac{9^{4/5}}{9^{3/5}} + 6^{-2/3}6^{-4/3}$$

<u>*Solutions*</u>.

We use the formulas

$$\boxed{\frac{a^m}{a^n} = a^{m-n}} \quad , \quad \boxed{a^m a^n = a^{m+n}} \tag{5}$$

Then

$$\frac{9^{4/5}}{9^{3/5}} + 6^{-2/3}6^{-4/3} = 9^{4/5-3/5} + 6^{-2/3-4/3} = 9^{\frac{4}{5}-\frac{3}{5}} + 6^{-\frac{2}{3}-\frac{4}{3}} = 9^{1/5} + 6^{-6/3} =$$

$$= 9^{1/5} + 6^{-2} \overset{(1),(3)}{=} \sqrt[5]{9} + \frac{1}{6^2} = \sqrt[5]{9} + \frac{1}{36} \quad . \quad \blacksquare$$

(3) Simplify.

$$\frac{\sqrt{98x^3}}{\sqrt{2x}} + \frac{\sqrt[3]{64}}{\sqrt[3]{8}}$$

<u>*Solutions*</u>.

Use (2) to simplify:

$$\frac{\sqrt{98x^3}}{\sqrt{2x}} + \frac{\sqrt[3]{64}}{\sqrt[3]{8}} = \sqrt{\frac{98x^3}{2x}} + \sqrt[3]{\frac{64}{8}} = \sqrt{\frac{\cancel{98}^{\,49} \cdot x^{\cancel{3}\,2}}{\cancel{2} \cdot \cancel{x}}} + \sqrt[3]{\frac{\cancel{64}^{\,8}}{\cancel{8}}} = \sqrt{49x^2} + \sqrt[3]{8} = 7|x| + 2 \ . \ \blacksquare$$

(4) Simplify and combine like radicals.

$$\sqrt{98} - \sqrt{50} - \sqrt{72}$$

<u>*Solutions*</u>.

We try to simplify to terms alike. First let us decompose 98, 50 and 72:

98	2
49	7
7	7
1	

50	2
25	5
5	5
1	

72	2
36	2
18	2
9	3
3	3
1	

Therefore, $98 = 2 \cdot 7^2 = 2 \cdot 49$, $50 = 2 \cdot 5^2 = 2 \cdot 25$, and $72 = 2^3 \cdot 3^2 = 2 \cdot 4 \cdot 9 = 2 \cdot 36$. Then

$$\sqrt{98} - \sqrt{50} - \sqrt{72} = \sqrt{2 \cdot 49} - \sqrt{2 \cdot 25} - \sqrt{2 \cdot 36} = 7\sqrt{2} - 5\sqrt{2} - 6\sqrt{2} = -4\sqrt{2} \ . \ \blacksquare$$