Math 109 P Section 2 — Quiz 8 B

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Solutions

(1) Simplify each expression.

(a) $9^{1/2}$ (b) $\left(\frac{1}{9}\right)^{1/2}$ (c) $(-64)^{1/3}$ (d) $\left(\frac{8}{27}\right)^{-4/3}$

Solutions.

(a) We have the formula

$$\boxed{a^{1/n} = \sqrt[n]{a}} \tag{1}$$

Then, according to (1): $9^{1/2} = \sqrt{9} = 3$. ■

(b) We have the formula

$$\boxed{\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}} \tag{2}$$

Then, according to (1) and (2) we have

$$\left(\frac{1}{9}\right)^{1/2} \overset{(1)}{=} \sqrt{\frac{1}{9}} \overset{(2)}{=} \frac{\sqrt{1}}{\sqrt{9}} = \frac{1}{3} \quad . \quad ■$$

(c) We use the formula (1):

$$(-64)^{1/3} = \sqrt[3]{-64} = -4 \quad . \quad ■$$

(d) We have the formulas

$$\boxed{\left(\frac{a}{b}\right)^{-m/n} = \left(\frac{b}{a}\right)^{m/n} = \left(\sqrt[n]{\frac{b}{a}}\right)^{m}} \tag{3}$$

and

$$\boxed{\left(\frac{a}{b}\right)^{n} = \frac{a^n}{b^n}} \tag{4}$$

Therefore, we use (3), (2), and (4):

$$\left(\frac{8}{27}\right)^{-4/3} \overset{(3)}{=} \left(\frac{27}{8}\right)^{4/3} = \left(\sqrt[3]{\frac{27}{8}}\right)^{4} \overset{(2)}{=} \left(\frac{\sqrt[3]{27}}{\sqrt[3]{8}}\right)^{4} = \left(\frac{3}{2}\right)^{4} \overset{(4)}{=} \frac{3^4}{2^4} = \frac{81}{16} \quad . \quad ■$$

(2) Perform the operations. Write your answer without negative exponents.

$$\frac{7^{2/3}}{7^{1/2}} + 5^{1/3}5^{-5/3}$$

<u>*Solutions*</u>.

We use the formulas

$$\boxed{\frac{a^m}{a^n} = a^{m-n}} \quad , \quad \boxed{a^m a^n = a^{m+n}} \tag{5}$$

Then

$$\frac{7^{2/3}}{7^{1/2}} + 5^{1/3}5^{-5/3} = 7^{2/3-1/2} + 5^{1/3-5/3} = 7^{\frac{4}{6}-\frac{3}{6}} + 5^{\frac{1}{3}-\frac{5}{3}} = 7^{1/6} + 5^{-4/3} =$$

$$= 7^{1/6} + \frac{1}{5^{4/3}} \overset{(1),(3)}{=} \sqrt[6]{7} + \frac{1}{\sqrt[3]{5^4}} = \sqrt[6]{7} + \frac{1}{5\sqrt[3]{5}} \quad . \quad \blacksquare$$

(3) Simplify.

$$\frac{\sqrt{75y^3}}{\sqrt{3y}} + \frac{\sqrt[3]{48}}{\sqrt[3]{6}}$$

<u>*Solutions*</u>.

Use (2) to simplify:

$$\frac{\sqrt{75y^3}}{\sqrt{3y}} + \frac{\sqrt[3]{48}}{\sqrt[3]{6}} = \sqrt{\frac{75y^3}{3y}} + \sqrt[3]{\frac{48}{6}} = \sqrt{\frac{\cancel{75}^{\,25} \cdot y^{\cancel{3}\,2}}{\cancel{3} \cdot \cancel{y}}} + \sqrt[3]{\frac{\cancel{48}^{\,8}}{\cancel{6}}} = \sqrt{25y^2} + \sqrt[3]{8} = 5|y| + 2 \ . \ \blacksquare$$

(4) Simplify and combine like radicals.

$$\sqrt{20} + \sqrt{125} - \sqrt{80}$$

<u>*Solutions*</u>.

We try to simplify to terms alike. First let us decompose 20, 125 and 80:

$$\begin{array}{r|l} 20 & 2 \\ 10 & 2 \\ 5 & 5 \\ 1 & \end{array} \qquad \begin{array}{r|l} 125 & 5 \\ 25 & 5 \\ 5 & 5 \\ 1 & \end{array} \qquad \begin{array}{r|l} 80 & 2 \\ 40 & 2 \\ 20 & 2 \\ 10 & 2 \\ 5 & 5 \\ 1 & \end{array}$$

Therefore, $20 = 2^2 \cdot 5 = 4 \cdot 5$, $125 = 5^3 = 25 \cdot 5$, and $80 = 2^4 \cdot 5 = 16 \cdot 5$. Then

$$\sqrt{20} + \sqrt{125} - \sqrt{80} = \sqrt{4 \cdot 5} + \sqrt{25 \cdot 5} - \sqrt{16 \cdot 5} = 2\sqrt{5} + 5\sqrt{5} - 4\sqrt{5} = 3\sqrt{5} \ . \ \blacksquare$$