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Solutions

(1) Perform the multiplication and simplify.

$$(a) \sqrt[3]{2} \cdot \sqrt[3]{12} \qquad (b) \sqrt[3]{3x^4y} \cdot \sqrt[3]{18x}$$

Solutions.

Recall the formulas

$$\sqrt[3]{a \cdot b} = \sqrt[3]{a} \cdot \sqrt[3]{b} \quad , \quad \sqrt[3]{a^3} = (\sqrt[3]{a})^3 = a \quad . \quad (1)$$

(a) Use (1), to simplify $\sqrt[3]{2} \cdot \sqrt[3]{12}$:

$$\sqrt[3]{2} \cdot \sqrt[3]{12} = \sqrt[3]{2 \cdot 12} = \sqrt[3]{2 \cdot 4 \cdot 3} = \sqrt[3]{8 \cdot 3} = \sqrt[3]{8} \cdot \sqrt[3]{3} = 2\sqrt[3]{3} \quad . \quad \blacksquare$$

(b) Use (1), to simplify $\sqrt[3]{3x^4y} \cdot \sqrt[3]{18x}$:

$$\begin{aligned} \sqrt[3]{3x^4y} \cdot \sqrt[3]{18x} &= \sqrt[3]{3x^4y \cdot 18x} = \sqrt[3]{54x^5y} = \sqrt[3]{27 \cdot 2 \cdot x^3 \cdot x^2 \cdot y} = \sqrt[3]{27 \cdot x^3 \cdot 2x^2y} = \\ &= \sqrt[3]{27} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{2x^2y} = 3x \sqrt[3]{2x^2y} \quad . \quad \blacksquare \end{aligned}$$

(2) Perform the operations and simplify.

$$(7\sqrt{x} + 2)(3\sqrt{x} - 5)$$

Solutions.

Distribute (perform the multiplication, foil):

$$\begin{aligned} (7\sqrt{x} + 2)(3\sqrt{x} - 5) &= 7\sqrt{x} \cdot 3\sqrt{x} - 7\sqrt{x} \cdot 5 + 2 \cdot 3\sqrt{x} - 2 \cdot 5 = 21(\sqrt{x})^2 - 35\sqrt{x} + 6\sqrt{x} - 10 = \\ &= 21 \underbrace{(\sqrt{x})^2}_{=x} - 35\sqrt{x} + 6\sqrt{x} - 10 = 21x - 29\sqrt{x} - 10 \quad . \quad \blacksquare \end{aligned}$$

(3) Rationalize the denominator. Simplify first.

$$\frac{\sqrt{10xy^2}}{\sqrt{2xy^3}}$$

Solutions.

We can use the following formula first:

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \quad . \quad (2)$$

Then, by (2):

$$\frac{\sqrt{10xy^2}}{\sqrt{2xy^3}} = \sqrt{\frac{10xy^2}{2xy^3}} = \sqrt{\frac{1\cancel{0}^5 \cdot \cancel{x} \cdot y^2}{\cancel{2} \cdot \cancel{x} \cdot y^3}} = \sqrt{\frac{5}{y}} \stackrel{(2)}{=} \frac{\sqrt{5}}{\sqrt{y}} = \frac{\sqrt{5} \cdot \sqrt{y}}{\sqrt{y} \cdot \sqrt{y}} = \frac{\sqrt{5} \cdot \sqrt{y}}{\sqrt{y} \cdot \sqrt{y}} = \frac{\sqrt{5y}}{y} \quad . \quad \blacksquare$$

(4) Solve and check the solution.

$$\sqrt{6x + 13} - 2 = 5$$

Solutions.

$$\text{Isolate the square root first : } \underbrace{\sqrt{6x + 13} - 2}_{+2} = \underbrace{5}_{+2} \Rightarrow \sqrt{6x + 13} = 7 \quad .$$

Then, raise both sides to power two, and solve:

$$\left(\sqrt{6x + 13}\right)^2 = 7^2 \Rightarrow 6x + 13 = 49 \Rightarrow \underbrace{6x + 13}_{-13} = \underbrace{49}_{-13} \Rightarrow 6x = 36 \Rightarrow \frac{6x}{6} = \frac{36}{6} \Rightarrow x = 6 \quad .$$

Check the solution :

$$6 \rightarrow \sqrt{6x + 13} - 2 = 5 \Rightarrow \sqrt{6 \cdot 6 + 13} - 2 = 5 \Rightarrow \sqrt{49} - 2 = 5 \Rightarrow 7 - 2 = 5 \Rightarrow \text{TRUE} \quad .$$

Therefore, $x = 6$ is the solution to the given equation . \blacksquare