

Figure 1: The graph of $f(x) = e^x$

3. Which answer describes the graph of the logarithmic function $f(x) = \ln x$?

- (a) The graph goes through $(0, 1)$ and has $x = 0$ as a vertical asymptote.
- (b) The graph goes through $(1, 0)$ and has $x = 0$ as a vertical asymptote.
- (c) The graph goes through $(0, 1)$ and has $y = 0$ as a horizontal asymptote.
- (d) The graph goes through $(1, 0)$ and has $y = 0$ as a horizontal asymptote.
- (e) The graph is a straight line through $(0, 1)$ and $(e, 1)$.

Solution :

The graph goes through $(1, 0)$ since $\ln 1 = 0$. It has a vertical asymptote at $x = 0$ (the y -axis).

The correct answer is b . ■

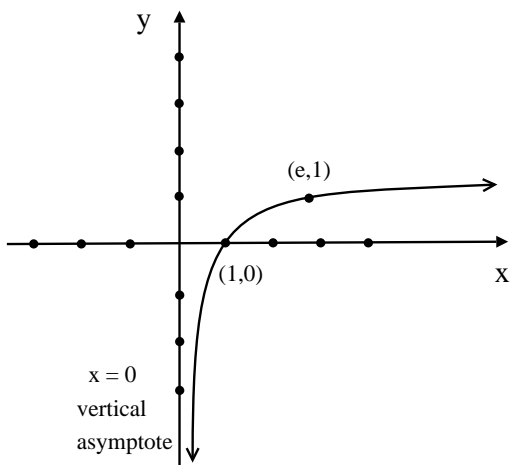


Figure 2: The graph of $f(x) = \ln x$

6. The function $f(x) = 3x - 2$ is a one-to-one function. Find the inverse function f^{-1} .

(a) $f^{-1}(x) = \frac{1}{3x - 2}$

(d) $f^{-1}(x) = \frac{1}{3}x + \frac{2}{3}$

(b) $f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$

(e) None of these

(c) $f^{-1}(x) = \frac{1}{2}x - \frac{3}{2}$

Solution :

Set $y = 3x - 2$, interchange x and y , and solve for y :

$$y = 3x - 2 \Rightarrow x = 3y - 2 \Rightarrow x + 2 = 3y \Rightarrow y = \frac{x + 2}{3} = \frac{x}{3} + \frac{2}{3} = \frac{1}{3}x + \frac{2}{3} .$$

The correct answer is d . ■

7. The function $f(x) = \frac{2}{x + 3}$ is a one-to-one function. Find the inverse function f^{-1} .

(a) $f^{-1}(x) = \frac{2}{x} - 3$

(d) $f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$

(b) $f^{-1}(x) = \frac{2}{x} - \frac{2}{3}$

(e) None of these

(c) $f^{-1}(x) = \frac{2}{x} + \frac{2}{3}$

Solution :

Set $y = \frac{2}{x + 3}$, interchange x and y , and solve for y :

$$\begin{aligned} y = \frac{2}{x + 3} \Rightarrow x = \frac{2}{y + 3} \Rightarrow x \cdot (y + 3) &= \frac{2}{y + 3} \cdot (y + 3) \Rightarrow \\ \Rightarrow xy + 3x = 2 \Rightarrow xy = 2 - 3x \Rightarrow y &= \frac{2 - 3x}{x} \Rightarrow y = \frac{2}{x} - \frac{3x}{x} \Rightarrow y = \frac{2}{x} - 3 . \end{aligned}$$

The correct answer is a . ■

8. Find the range of the function $f(x) = \frac{2}{x+3}$. (See the previous problem)

- (a) $\{y \mid y \neq 0\}$ (d) All real numbers
(b) $\{y \mid y \neq 2/3\}$ (e) None of these
(c) $\{y \mid y \neq 1/2\}$

Solution :

By the previous exercise, the inverse function of $f(x) = \frac{2}{x+3}$ is $f^{-1}(x) = \frac{2-3x}{x}$.

We know that the range of our function f is the same as the domain of its inverse, f^{-1} . The domain of $f^{-1}(x)$ consists of all real numbers except $x = 0$. Therefore, the range of f is all reals except $y = 0$ (because you interchange x and y).

The correct answer is a. ■

9. Find the domain of $f \circ g$ if $f(x) = \frac{1}{2x-1}$ and $g(x) = \frac{1}{x+1}$.

- (a) $\{x \mid x \neq 1\}$ (d) $\{x \mid x \neq 1/2\}$
(b) $\{x \mid x \neq -1\}$ (e) None of these
(c) $\{x \mid x \neq 1 \text{ and } x \neq -1\}$

Solution :

By definition, $(f \circ g)(x) = f(g(x))$. Therefore, the domain of g affects the domain of f . Since

$$g(x) = \frac{1}{x+1} \Rightarrow \text{the domain of } g \text{ is } \{x \mid x \neq -1\}.$$

Now compute $f \circ g$:

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) = f\left(\frac{1}{x+1}\right) = \frac{1}{2 \cdot \frac{1}{x+1} - 1} = \frac{1}{2 \cdot \frac{1}{x+1} - 1} \cdot \frac{x+1}{x+1} = \\ &= \frac{x+1}{2 \cdot \frac{1}{x+1} \cdot (x+1) - (x+1)} = \frac{x+1}{2-x-1} = \frac{x+1}{1-x}. \end{aligned}$$

Therefore, we have another point to exclude : $x \neq 1$.

The correct answer is c. ■

12. Solve for x : $\sqrt{3}^{x+2} = \frac{1}{9}$

(a) $x = -4$

(d) $x = -3/2$

(b) $x = -5$

(e) None of these

(c) $x = -6$

Solution :

We can write everything in base 3 :

$$\sqrt{3}^{x+2} = \frac{1}{9} \Rightarrow \left(3^{\frac{1}{2}}\right)^{x+2} = \frac{1}{3^2} \Rightarrow 3^{\frac{1}{2}(x+2)} = 3^{-2} \Rightarrow \frac{1}{2}(x+2) = -2 \Rightarrow x+2 = -4 \Rightarrow x = -6 .$$

The correct answer is c . ■

13. If $f(x) = \log_5(x)$, what is $f^{-1}(x)$?

(a) $f^{-1}(x) = -\log_5(x)$

(b) $f^{-1}(x) = \frac{1}{\log_5(x)}$

(c) $f^{-1}(x) = \log_{\frac{1}{5}}(x)$

(d) $f^{-1}(x) = 5^x$

(e) f^{-1} does not exist

Solution :

$f(x) = \log_5(x)$ and $f^{-1}(x) = 5^x$ are inverse to each other. Here is how we can find the inverse of f :

$$f(x) = \log_5(x) = y \xleftrightarrow{\text{change } x \leftrightarrow y} \log_5(y) = x \Rightarrow 5^x = y \Rightarrow f^{-1}(x) = 5^x .$$

The correct answer is d . ■

14. The domain of $f(x) = \log(1 - 5x)$ is

(a) $\left(\frac{1}{5}, \infty\right)$

(d) $\left(-\infty, \frac{1}{5}\right]$

(b) $\left[\frac{1}{5}, \infty\right)$

(e) None of these

(c) $\left(-\infty, \frac{1}{5}\right)$

Solution :

The logarithm must be defined, so

$$f(x) = \log(1 - 5x) \Rightarrow 1 - 5x > 0 \Rightarrow 1 > 5x \Rightarrow 5x < 1 \Rightarrow x < \frac{1}{5} .$$

The correct answer is c . ■

15. $\log \left(\frac{x^3 \sqrt{x+1}}{(x-2)^2} \right) =$

- (a) $3 \log x + \frac{1}{2} \log(x+1) - 2 \log(x-2)$
- (b) $3 \log x + \frac{1}{2} \log(x+1) + 2 \log(x-2)$
- (c) $3 \log x + \log(x+1) - \log(x-2)$
- (d) $3 \log x + \log(x+1) + \log(x-2)$
- (e) None of these

Solution :

We use the properties of the logarithms :

$$\log M \cdot N = \log M + \log N, \quad \log \frac{M}{N} = \log M - \log N, \quad \text{and} \quad \log M^c = c \cdot \log M.$$

Then

$$\begin{aligned} \log \left(\frac{x^3 \sqrt{x+1}}{(x-2)^2} \right) &= \log(x^3 \sqrt{x+1}) - \log(x-2)^2 = \log(x^3) + \log(\sqrt{x+1}) - 2 \log(x-2) = \\ &= 3 \log x + \log(x+1)^{\frac{1}{2}} - 2 \log(x-2) = 3 \log x + \frac{1}{2} \log(x+1) - 2 \log(x-2). \end{aligned}$$

The correct answer is a . ■

16. Express y as a function of x (the constant C is positive). $\ln y = 3x + \ln C$

- (a) $y = \ln(3x) + C$
- (b) $y = Ce^{3x}$
- (c) $y = C^{3x}$
- (d) $y = e^{3x} + C$
- (e) None of these

Solution :

Move the natural logarithms on one side, and use the properties, and the connection with the exponential function :

$$\ln y - \ln C = 3x \Rightarrow \ln \frac{y}{C} = 3x \Rightarrow e^{3x} = \frac{y}{C} \Rightarrow y = Ce^{3x} .$$

The correct answer is b . ■

17. Solve for x : $\ln(x + 1) + \ln(x) = \ln(6)$

(a) $x = -3$ or $x = 2$

(d) $x = 1/5$

(b) $x = 2$

(e) None of these

(c) $x = -3$

Solution :

Using the logarithm properties :

$$\begin{aligned}\ln(x + 1) + \ln(x) = \ln(6) &\Rightarrow \ln x(x + 1) = \ln(6) \Rightarrow x(x + 1) = 6 \Rightarrow \\ &\Rightarrow x^2 + x - 6 = 0 \Rightarrow (x - 2)(x + 3) = 0 \Rightarrow x = 2 \text{ or } x = -3 \quad .\end{aligned}$$

But $x = -3$ cannot be accepted; for example, $\log x \mapsto \log(-3)$ is not defined. The solution is $x = 2$ only.

The correct answer is b . ■

18. Solve for x : $2^{x+1} = 6$

(a) $x = \ln 3$

(d) $x = \frac{\ln 6}{\ln 2} - 1$

(b) $x = \ln 4$

(e) None of these

(c) $x = \ln 4 - \ln 2$

Solution :

$$2^{x+1} = 6 \Rightarrow \ln 2^{x+1} = \ln 6 \Rightarrow (x + 1) \ln 2 = \ln 6 \Rightarrow x + 1 = \frac{\ln 6}{\ln 2} \Rightarrow x = \frac{\ln 6}{\ln 2} - 1 \quad .$$

The correct answer is d . ■

19. $(\log_3 6)(\log_6 9) =$

(a) $x = 2$

(d) $x = \log_3(3/2)$

(b) $x = 3$

(e) None of these

(c) $x = \log_6 3$

Solution :

Use the formula :

$$\log_a b = \frac{\log_b b}{\log_b a} = \frac{1}{\log_b a} \Rightarrow \log_3 6 = \frac{1}{\log_6 3} \quad .$$

Therefore,

$$(\log_3 6)(\log_6 9) = \left(\frac{1}{\log_6 3} \right) (\log_6 9) = \frac{1}{\log_6 3} \cdot \log_6 3^2 = \frac{1}{\log_6 3} \cdot 2 \cdot \log_6 3 = 2 \cdot \frac{1}{\log_6 3} \cdot \log_6 3 = 2 .$$

The correct answer is a . ■

20. Solve for x : $5^x = 3^{1-2x}$

(a) $x = 3/7$

(d) $x = \frac{1}{\ln 5 + 2 \ln 3}$

(b) $x = 11/7$

(e) None of these

(c) $x = \frac{\ln 3}{\ln 5 + 2 \ln 3}$

Solution :

$$\begin{aligned} 5^x = 3^{1-2x} &\Rightarrow \ln 5^x = \ln 3^{1-2x} \Rightarrow x \cdot \ln 5 = (1 - 2x) \cdot \ln 3 \Rightarrow x \cdot \ln 5 = \ln 3 - 2x \ln 3 \Rightarrow \\ &\Rightarrow x \cdot \ln 5 + 2x \ln 3 = \ln 3 \Rightarrow x(\ln 5 + 2 \ln 3) = \ln 3 \Rightarrow x = \frac{\ln 3}{\ln 5 + 2 \ln 3} . \end{aligned}$$

The correct answer is c . ■