Math 229, Section 9 — Examen 1

Instructor: Racovitan Mihai **09/22/06**

Name _____ **Z-number** _____

Instructions: Answer all questions in the space provided. If you need extra space, use the back of the paper. No scratch paper is allowed. You must show all work in order to receive full credit. No calculators will be allowed. You have 9 exercises, labeled Ex. 1 to Ex. 9 .

Ex. 1) Let $f(x) = x + \frac{1}{x}$ and $g(x) = \frac{x+1}{x+2}$.

(a) (6 pts.) Find $f \circ g$.

(b) (6 pts.) Find the domain of $f \circ g$.

Ex. 2) (8 pts.) Express the following expression in the form $H(x) = (f \circ g)(x)$.

$$H(x) = \sin^4\left(\sqrt{x}\right)$$

Emphasize the functions f and g . Check your answer . Find the interval on which the function $H(x)$ is continuous at each point .

Ex. 3) (3 pts. each) Circle the appropriate answer . No explanation needed .

True False (a) The function $f(x) = \dfrac{x^2 - 1}{x + 1}$ is the same as the function $g(x) = x - 1$.

True False (b) The limit of the function $f(x) = \dfrac{x^2 - 1}{x + 1}$ is the same as the limit of the function $g(x) = x - 1$ when x approaches -1 .

Ex. 4) (a) (4 pts.) State the (ε, δ)-definition of the limit $\lim_{x \to a} f(x) = L$.

(b) (4 pts.) Use the (ε, δ)-definition of the limit to prove the following limit .

$$\lim_{x \to 1}(4x - 5) = -1$$

Ex. 5) (4 pts. each) Let $f(x)$ be a function .

(a) State the difference quotient definition of the derivative of $f(x)$.

(b) Explain how one can use $f'(a)$ to find the equation of the tangent line to the curve at a point $(a, f(a))$.

Ex. 6) (6 pts. each) Evaluate each of the following limits. You must show how you evaluated the limit (the simplifications and the laws) to get full credit. Do not use the (ε, δ)-definition of the limit.

(a) $\lim\limits_{x \to 5} \dfrac{x^2 - 2x - 15}{x - 5}$

(b) $\lim\limits_{h \to 0} \dfrac{\sqrt{h+4} - 2}{h}$

(c) $\lim\limits_{x \to 3} \dfrac{2x - 3}{x^2 - 3x + 2}$

(d) $\lim\limits_{x \to 0} \dfrac{2}{x^2(x-1)}$

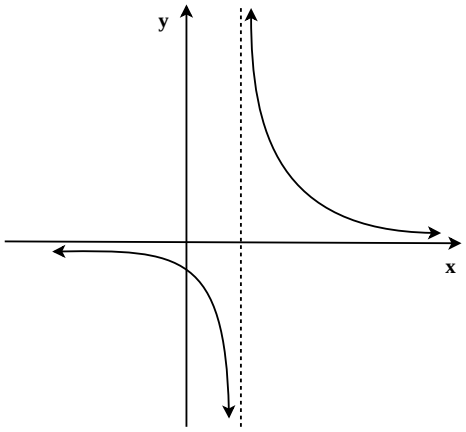
Ex. 7) Let $f(x) = \dfrac{x}{x+1}$.

(a) (8 pts.) Find the derivative $f'(x)$ by using the difference quotient definition of the derivative of $f(x)$.

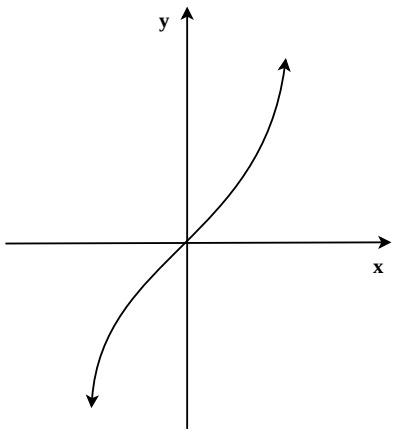
(b) (6 pts.) Find an equation of the tangent line to the curve $y = f(x)$ at the point $(1, \dfrac{1}{2})$.

Ex. 8) (8 pts.) Let $f(x) = \sqrt{2x-2} - 2x^2 + 3$. Show that there is a root of $f(x)$ (i.e. a value of x with the property that $f(x) = 0$) on the interval $[1, 3]$.

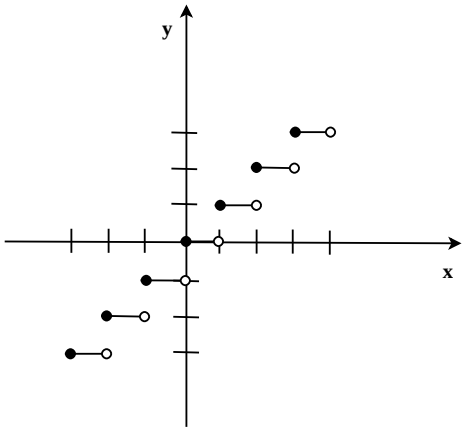
Ex. 9) (3 pts. each) Match the following graphs to the choice that fits the best.



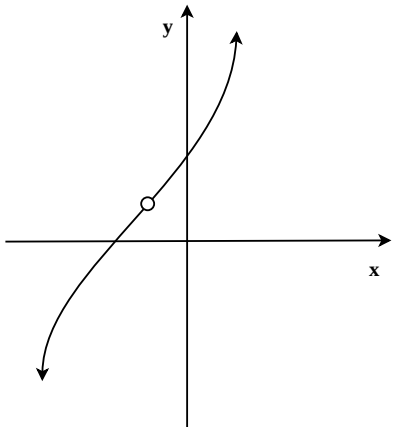
a) _____



b) _____



c) _____



d) _____

i) continuous everywhere

ii) jump discontinuity

iii) infinite discontinuity

iv) removable discontinuity

v) essential discontinuity (other than infinite discontinuity)

vi) nothing applies