Math 229, Section 9 <u>Examen 2</u>

Instructor: Racovitan Mihai **10/27/06**

Name __________________ **Z-number ____________**

Instructions: Answer all questions in the space provided. If you need extra space, use the back of the paper. No scratch paper is allowed. You must show <u>all work</u> in order to receive full credit. No calculators will be allowed. You have 8 exercises, labeled (Ex. 1) to (Ex. 8) .

(Ex. 1) (12 pts., 2 pts. each) State the following formulas, rules, and identities.

(a) The Chain Rule

(b) The trigonometric identity for $\sin(x - y)$

(c) The trigonometric identity that relates $\sec x$ and $\tan x$

(d) $\dfrac{d}{dx}(\cot x)$

(e) $\dfrac{d}{dx}(\sec x)$

(f) The Quotient Rule

(Ex. 2) (8 pts., 2 pts. each) Assume $0 < \theta < \dfrac{\pi}{2}$ and $\sin\theta = \dfrac{1}{3}$. Use trigonometric identities if needed to find the following.

(a) Find $\cos\theta$

(b) Find $\sin 2\theta$

(c) Find $\cos 2\theta$

(d) Find $\tan 2\theta$

(Ex. 3) (30 pts., 5 pts. each) Find the following derivatives. You don't need to simplify. Use shortcuts, not the definition of the derivative as a limit. State the rules you use (like Chain Rule, Product Rule, etc.)

(a) $f(x) = (2x+1)^{10} \cdot (3x^2 - 2x + 1)^{12}$

(b) $y = \dfrac{\sin^2 x}{\cos x} = \dfrac{(\sin x)^2}{\cos x}$

(c) $f(x) = \tan(3x^2 + 5)$

(d) $g(x) = \dfrac{x+2}{\sqrt{x}}$

(e) $y = x \cdot \sin x$

(f) Find $\dfrac{dy}{dx}$ for $x^3 + y^3 = 6xy$

(Ex. 4) (12 pts., 4 pts. each) For the function $f(x) = 2x^3 - 5x^2 + 4x - 7$

(a) Find the intervals where $f(x)$ is increasing or decreasing.

(b) Determine relative maximum and minimum values of the function and at which values of x these occur. Use either **First** or **Second Derivative Test**.

(c) Determine the intervals of concavity.

(Ex. 5) (10 pts.) The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2 ?

(Ex. 6) (10 pts.) Find the linear approximation of the function $g(x) = \sqrt[3]{1+x}$ at $a = 0$ and use it to approximate $\sqrt[3]{1.05}$.

(Ex. 7) (6 pts.) Find the absolute maximum and the absolute minimum of $f(x) = x + 2\cos x$ on the closed interval $\left[0, \frac{\pi}{2}\right]$.

(Ex. 8) (12 pts., 4 pts. each) Let $f(x) = x^5 + 8x + 1$.

(a) State the **Rolle's Theorem** .

(b) Use the **Intermediate Value Theorem** to show that $f(x)$ has at least one root (i.e. a number c in the domain of $f(x)$ such that $f(c) = 0$).

(c) Use the **Rolle's Theorem** to show that $f(x)$ has at most one root .