

# Math 229, Section 9

# Home take Examen B

Instructor: Racovitan Mihai

Due: Monday, 10/30/06

Name \_\_\_\_\_

Z-number \_\_\_\_\_

**Instructions:** Answer all questions in the space provided. You must show all work in order to receive full credit.

**(Ex. 1)** (12 pts., 4 pts. each) Solve the following. Show what rules, definitions or formulas you use.

(a) Find the limit  $\lim_{x \rightarrow 0} \frac{\sin(4x)}{\tan(3x)}$ .

(b) Find the trigonometric identity for  $\cos(3x)$  as an expression of  $\cos x$  and powers of  $\cos x$  only.

(c) Find all values of  $x$  in the interval  $[0, 2\pi]$  that satisfy the inequality.

$$\cos x > -\frac{1}{2}$$

**(Ex. 2)** (8 pts.) Assume  $0 < \theta < \frac{\pi}{2}$  and  $\tan \theta = \sqrt{3}$ . Find the remaining trigonometric ratios:

$\sin \theta$  ,  $\cos \theta$  ,  $\cot \theta$  ,  $\sec \theta$  ,  $\csc \theta$  (use trigonometric identities if needed)

**(Ex. 3)** (20 pts., 5 pts. each) Find the following derivatives. You don't need to simplify. Use shortcuts, not the definition of the derivative as a limit. State the rules you use (like Chain Rule, Product Rule, etc.)

(a) Find  $\frac{dy}{dx}$  for  $\tan(x \cdot y) = x - y^2$ .

(b) Find  $\frac{d^2y}{dx^2}$  (the second derivative) for  $\sqrt{y} = 2 + \sqrt{2x}$ .

(c) Find the 107<sup>th</sup> derivative of  $\cos x$  :  $D^{(107)} \cos x$  .

(d) Find  $y'$  for  $y = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x^2}}$

**(Ex. 4)** (12 pts., 4 pts. each) For the function  $f(x) = 3 - 5x + x^3$

(a) Find the intervals where  $f(x)$  is increasing or decreasing.

(b) Determine relative maximum and minimum values of the function and at which values of  $x$  these occur. Use either **First** or **Second Derivative Test**.

(c) Determine the intervals of concavity and the inflection points.

**(Ex. 5)** (10 pts.) A particle is moving along the curve  $y = 2\sqrt{x}$ . As the particle passes through the point  $(4, 4)$ , its  $x$ -coordinate increases at a rate of 3 cm/s. How fast is the distance from the particle to origin changing at this instant? (Hint : is not the rate at which the  $y$ -coordinate is changing. Drawing a picture would help.)

**(Ex. 6)** (10 pts.) Find the linear approximation of the function  $g(x) = \frac{1}{\sqrt{4+x}}$  at  $a = 0$  and use it to approximate  $\frac{1}{\sqrt{3.95}}$ .

**(Ex. 7)** (6 pts.) Find the absolute maximum and the absolute minimum of  $f(x) = x - 2 \cos x$  on the closed interval  $[0, \pi]$ .

(Ex. 8) (12 pts.) Show that the equation  $2x - 1 - \sin x = 0$  has exactly one real solution. (Hint: use the **Intermediate Value Theorem**, and the **Mean Value Theorem** or **Rolle's Theorem**).

(Ex. 9) (10 pts.) Find the equation of the tangent line to the curve  $\sqrt[3]{x} + \sqrt{y} = 1$  at the point  $(1, 0)$ .