Hat Problems

1. Easy Hat Problem

Three people are sent into a room and sit in a column facing forward. Each person is told that he or she will be given a hat from a stack of 3 blue hats and 2 red hats. Each person can only see the hat of the person or persons directly in front of them.

The third person in the row is asked “Do you know the color of your hat?”

“No” is the response.

The middle person is asked the same question and after some deliberation he also replies “No.”

When the first person in the column is asked if she knows the color of her hat, she thinks for a while, and then says “Yes, I do. My hat is blue.”

How does she know this?

2. Challenge Hat Problem

Three people are sent into a room, each wearing a hat whose color (red or blue) they cannot see.

(They can see the hats of the other two people.)

Once in the room, each person must write on a slip of paper one of three possible sentences:
1. I am wearing a \textcolor{red}{red} hat,
2. I am wearing a \textcolor{blue}{blue} hat, or

A correct response for the group of 3 people (worth a $1,000,000 prize) occurs if one of the three guesses correctly and the other two do not make an error.

The three people can decide on a strategy before they are given their hats.

What is the optimal strategy? What is their chance of winning?